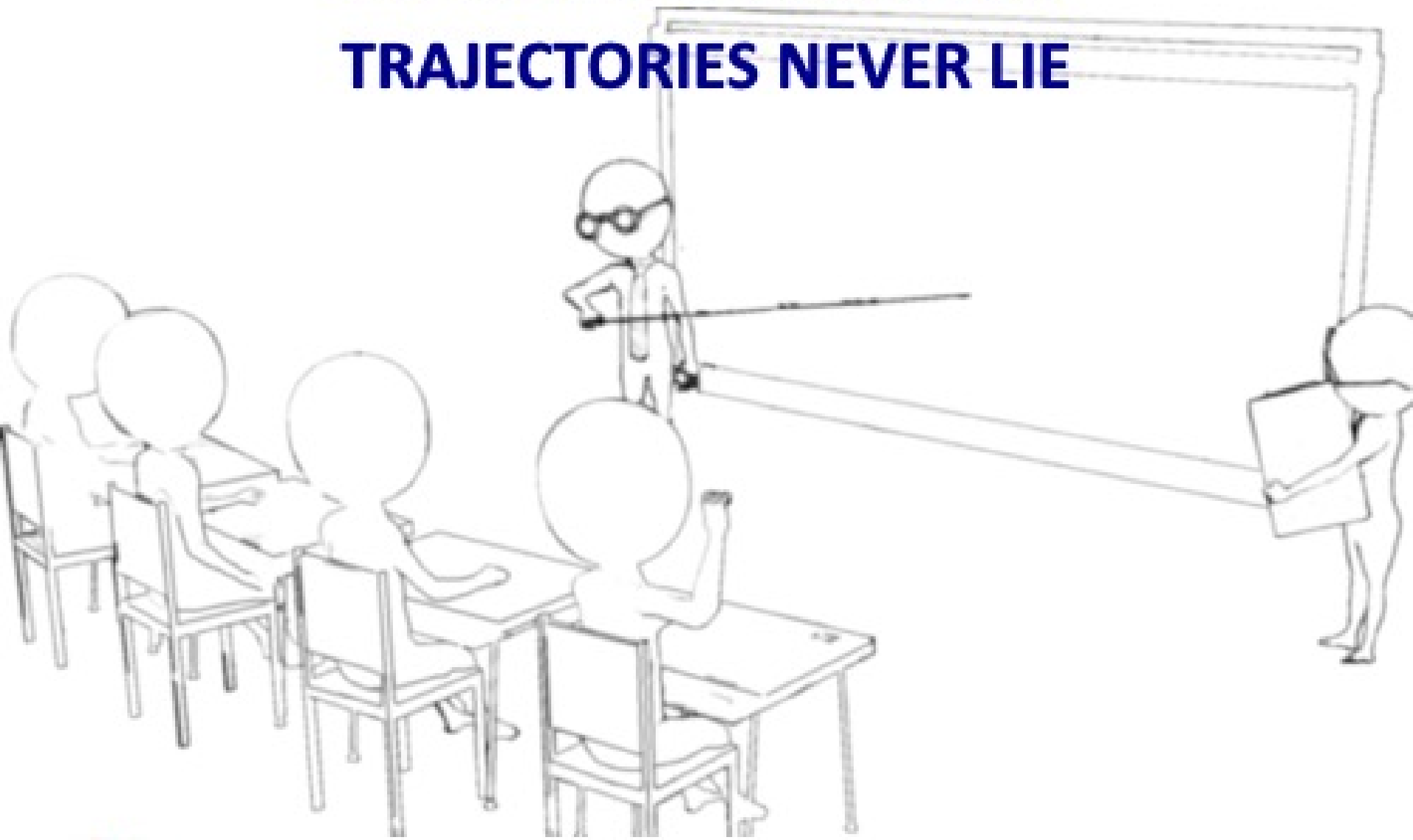


ACSR 2019 CONFERENCE

TRAJECTORIES NEVER LIE



GRAFF Investigative & Forensic Training

TRAJECTORIES NEVER LIE

overview

- The math behind analysis.
 - Evaluating the data: placing the shooter & victim
- Trajectories from ricochets.
- Ricochet angles in sheet metal
- Elevation and Azimuth angle protocol.
- Practical Exercises



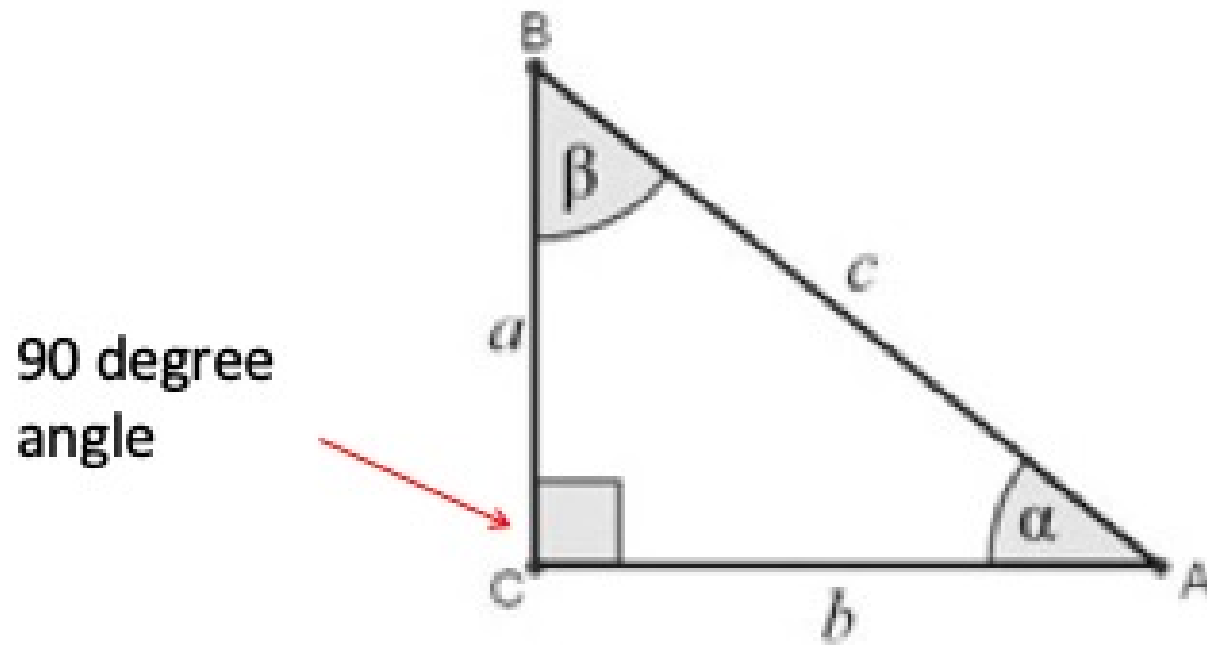
GRAFF Investigative & Forensic Training

Application of Trigonometry to Trajectory Analysis

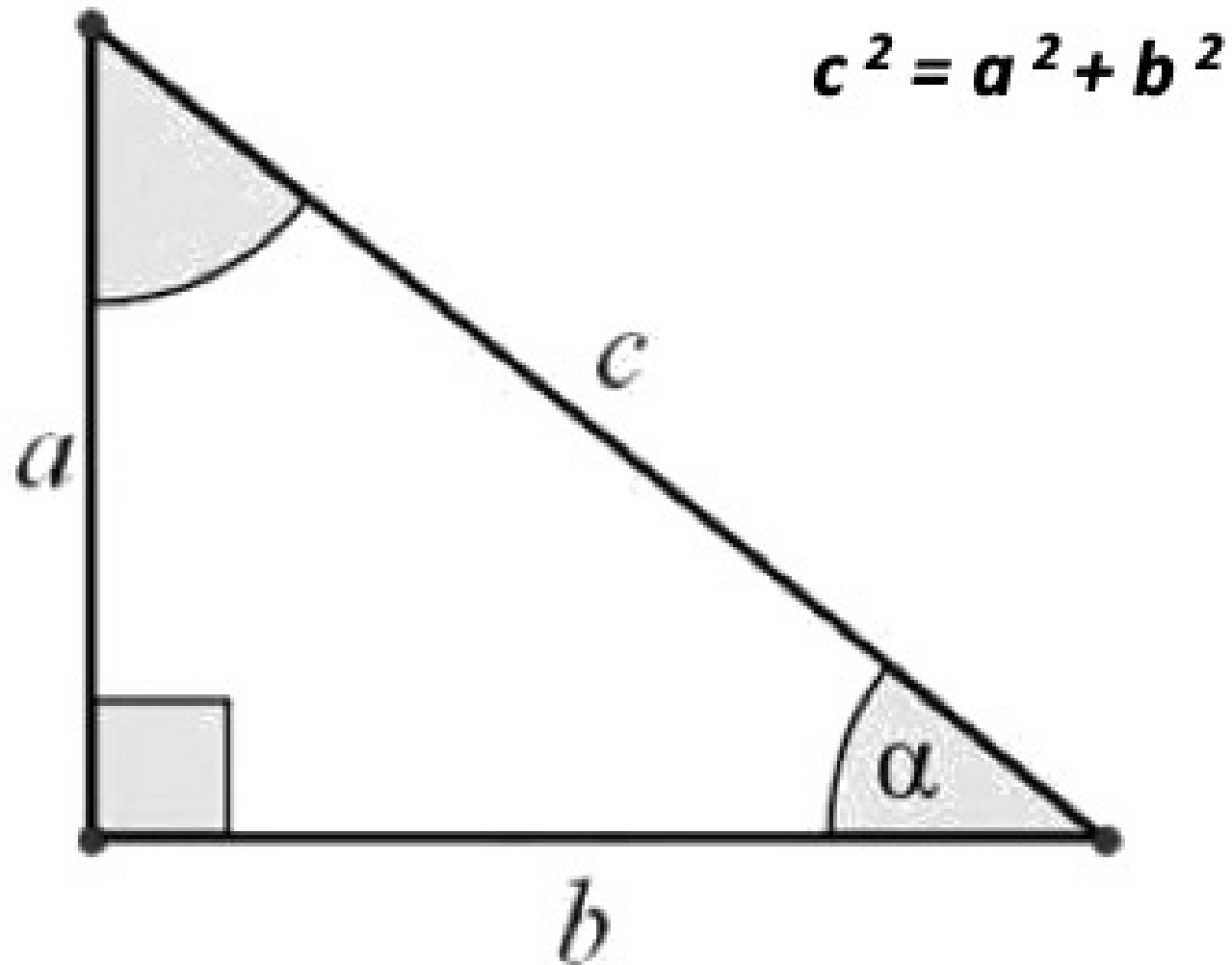
A triangles has six components.

A right triangles has one 90 degree angle.

If any 2 other components of the triangle are known, the other 3 components can be calculated.



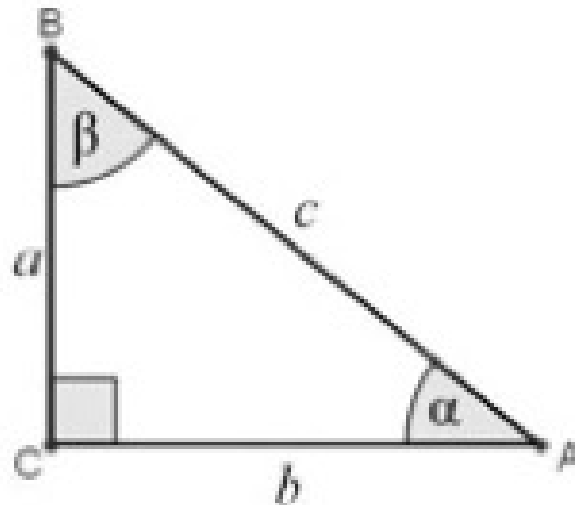
If two sides are known, the third side can be calculated:



This is known as Pythagorean Theorem

Application of Trigonometry to Trajectory Analysis

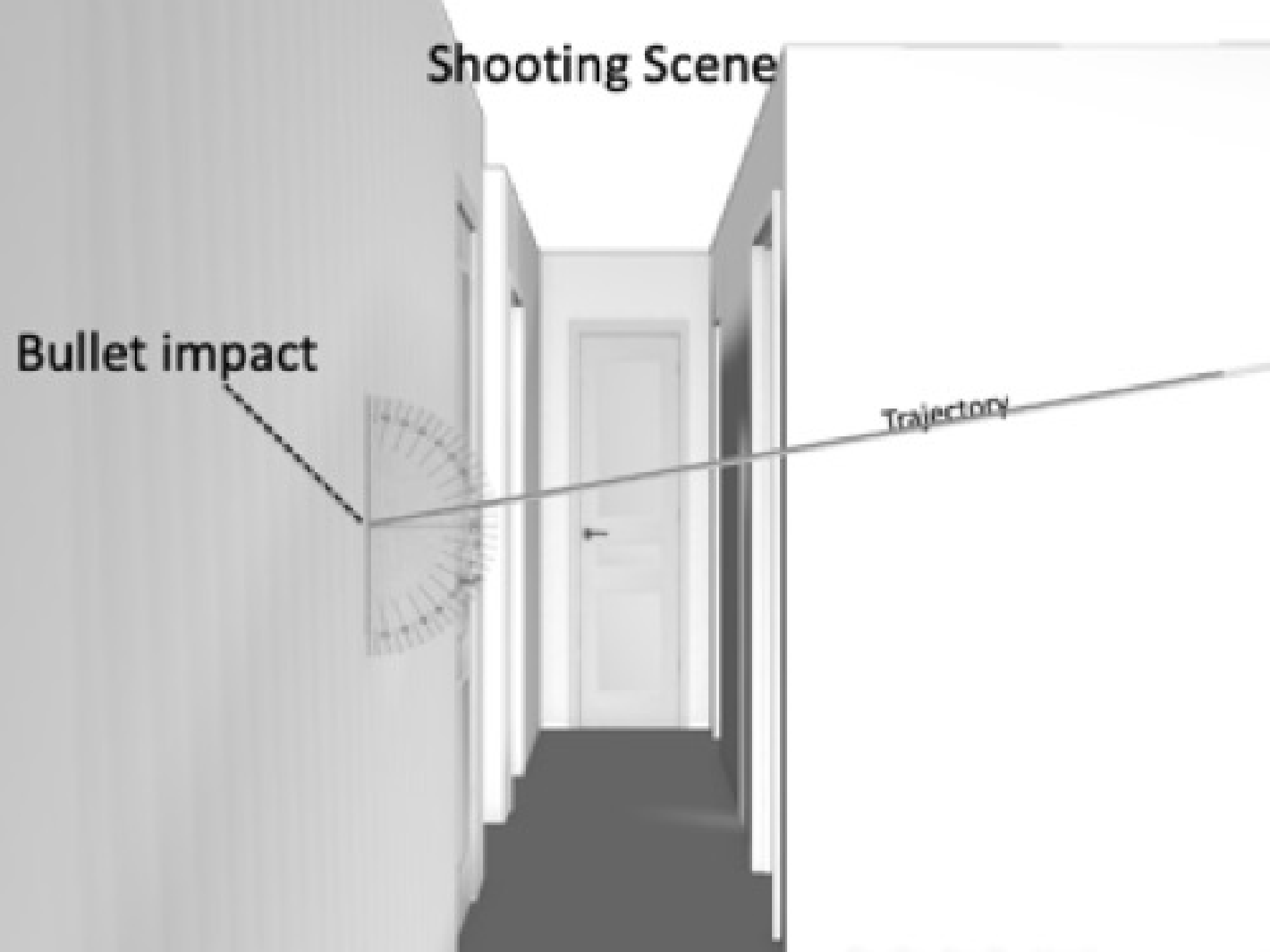
- Key to success: Applying right triangle to the scene.



Shooting Scene

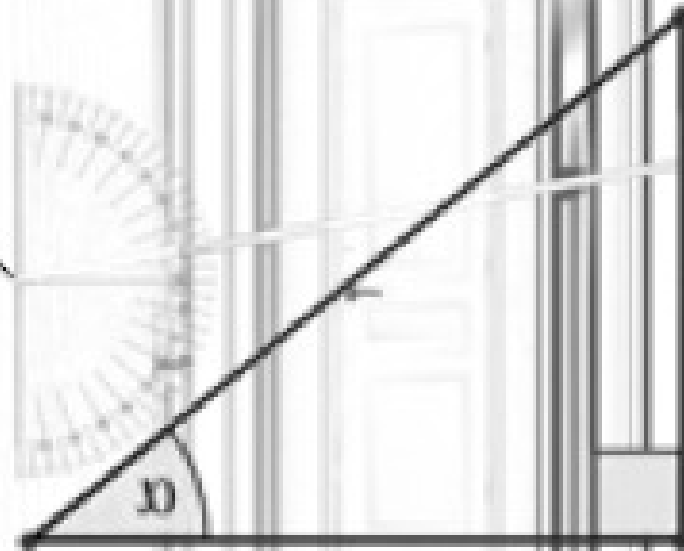
Bullet impact

Trajectory



To apply the triangle in your scene:
 α angle is always the angle taken
at bullet impact point.

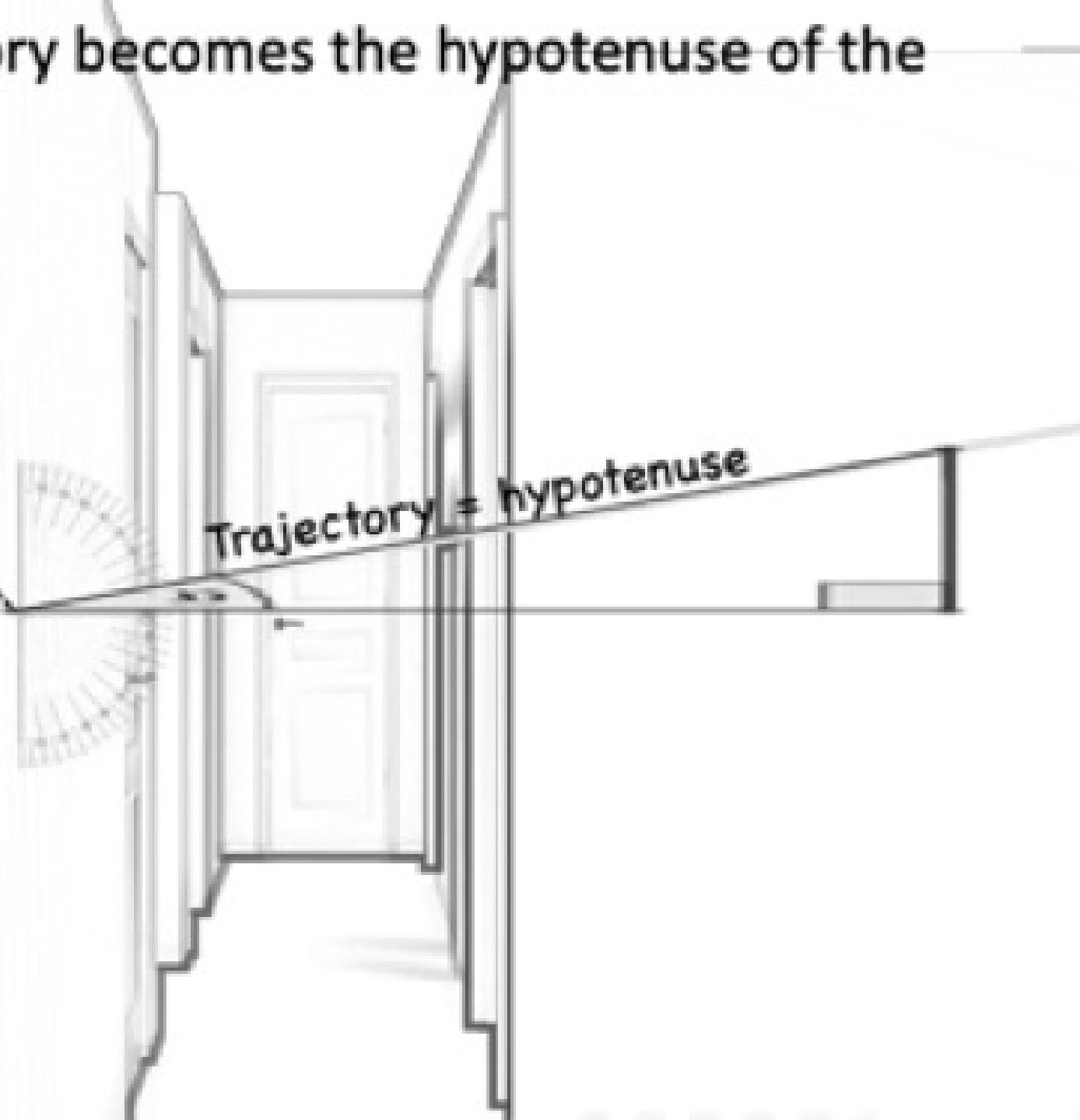
Bullet impact



The trajectory becomes the hypotenuse of the triangle.

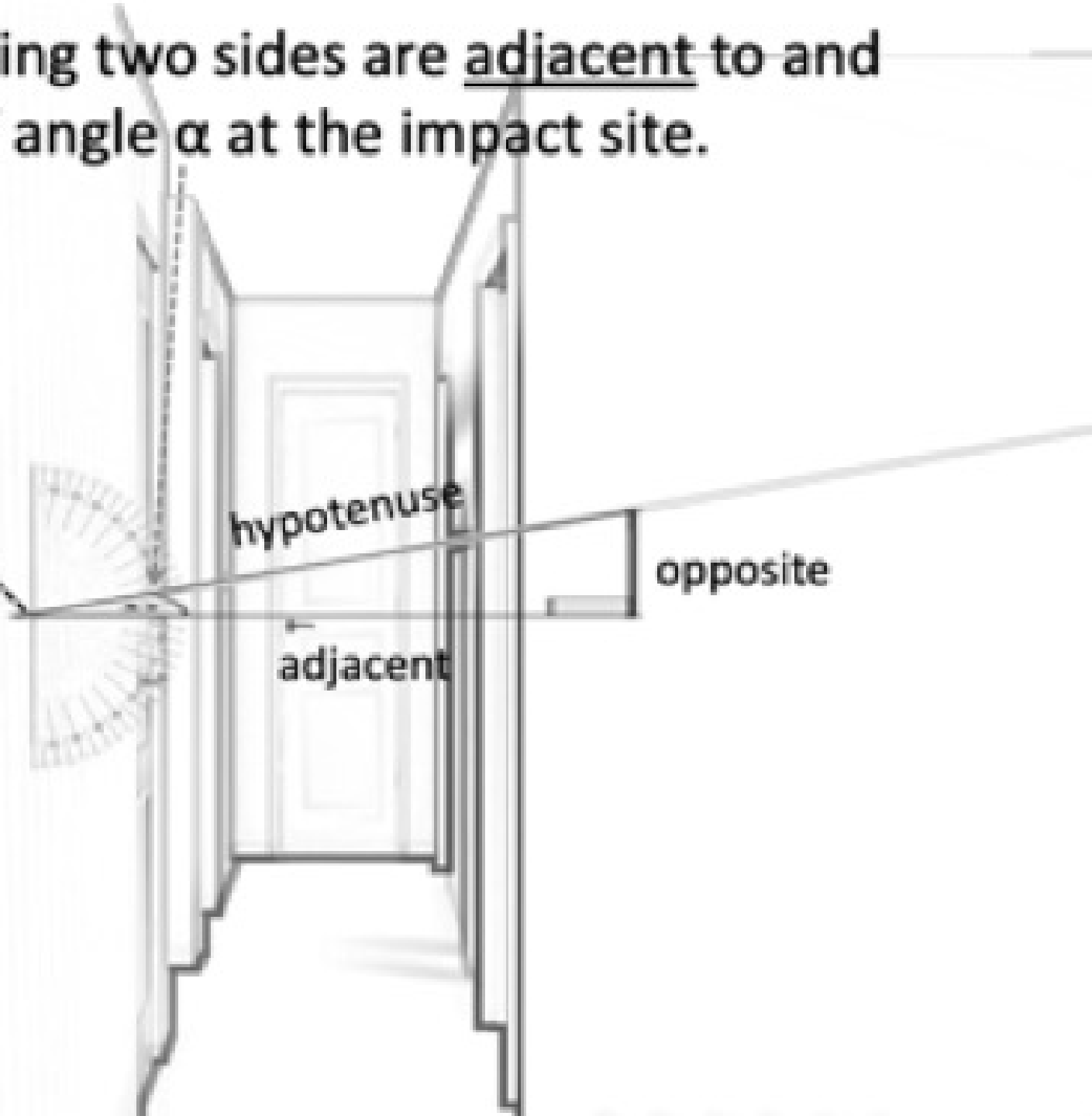
Bullet impact

Trajectory = hypotenuse



The remaining two sides are adjacent to and opposite of angle α at the impact site.

Bullet impact



In shooting scenes, parts of the triangle often relate to the event as follows:

α is *elevation or azimuth angle*

Hypotenuse = *trajectory*

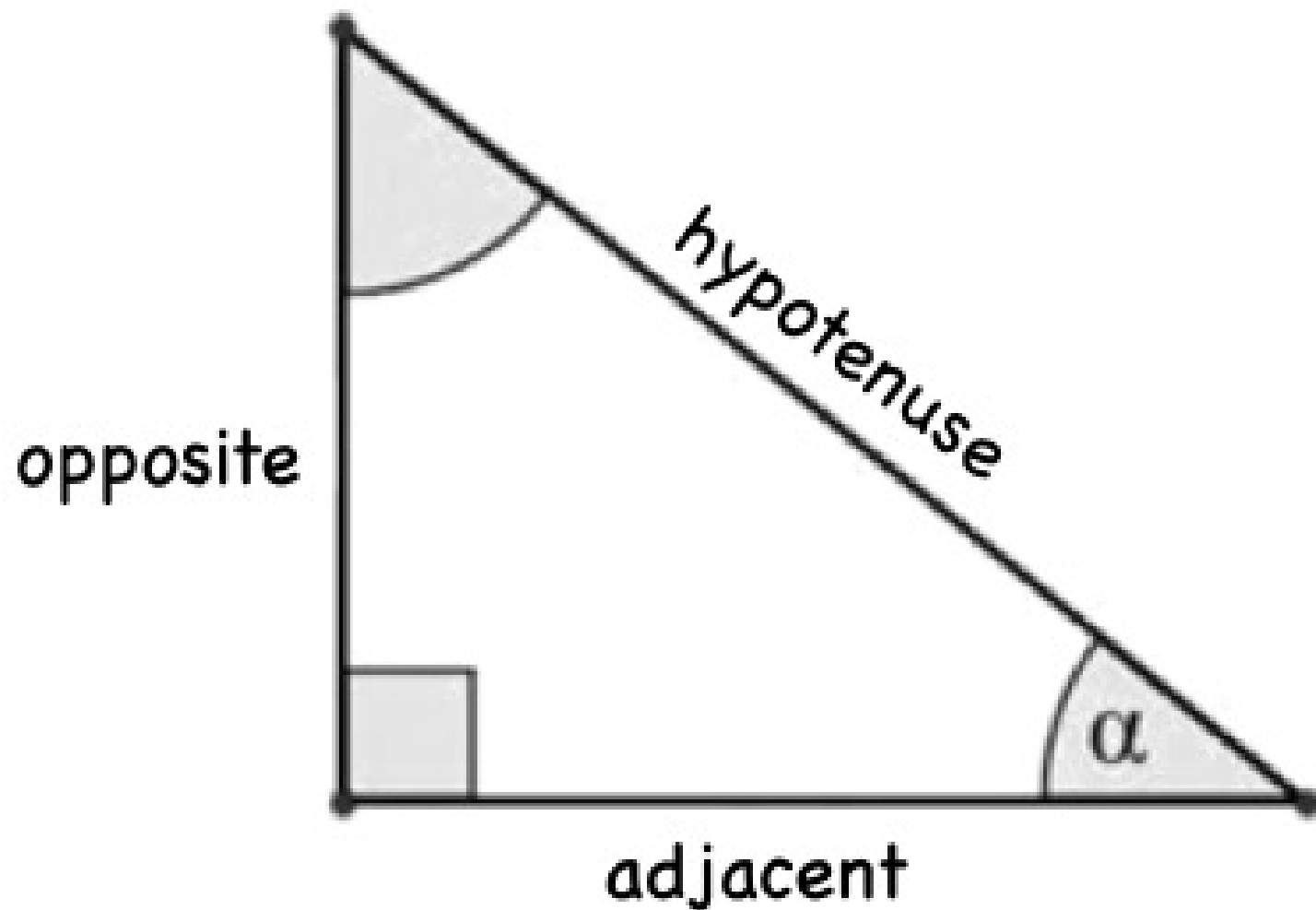
Adjacent side = *distance to shooter (muzzle)*

Opposite side = *height of muzzle above/below impact or lateral distance (azimuth calculation)*

Bullet impact

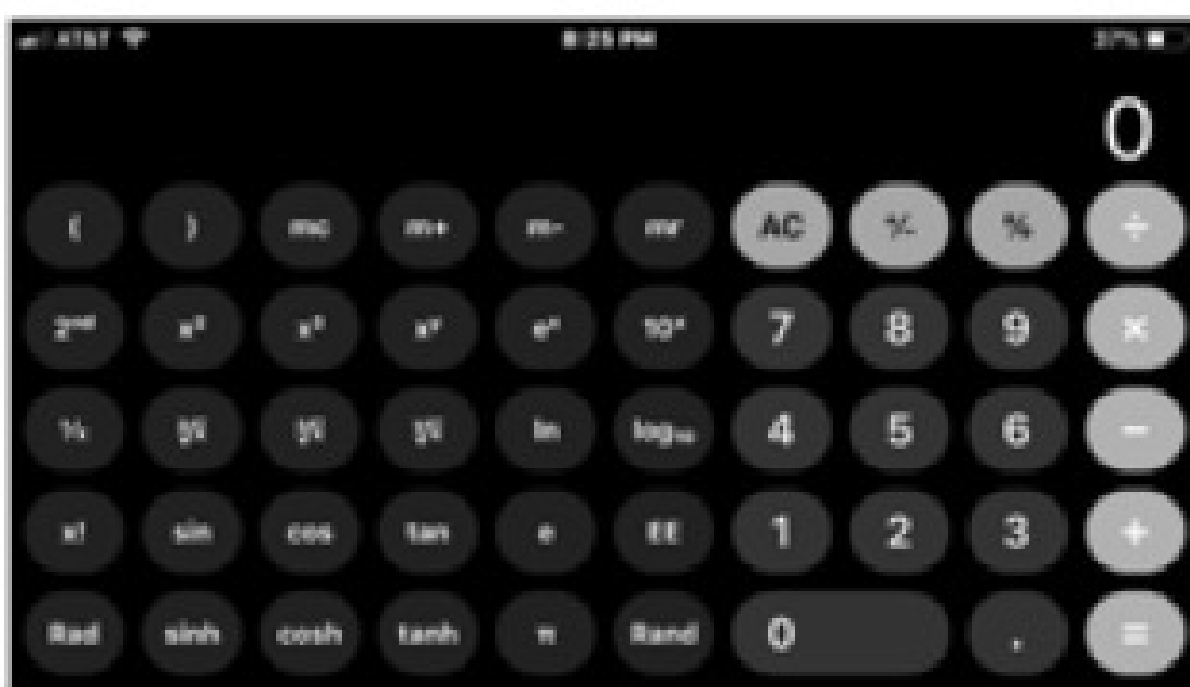


Relationships of Angles to Sides of Triangle



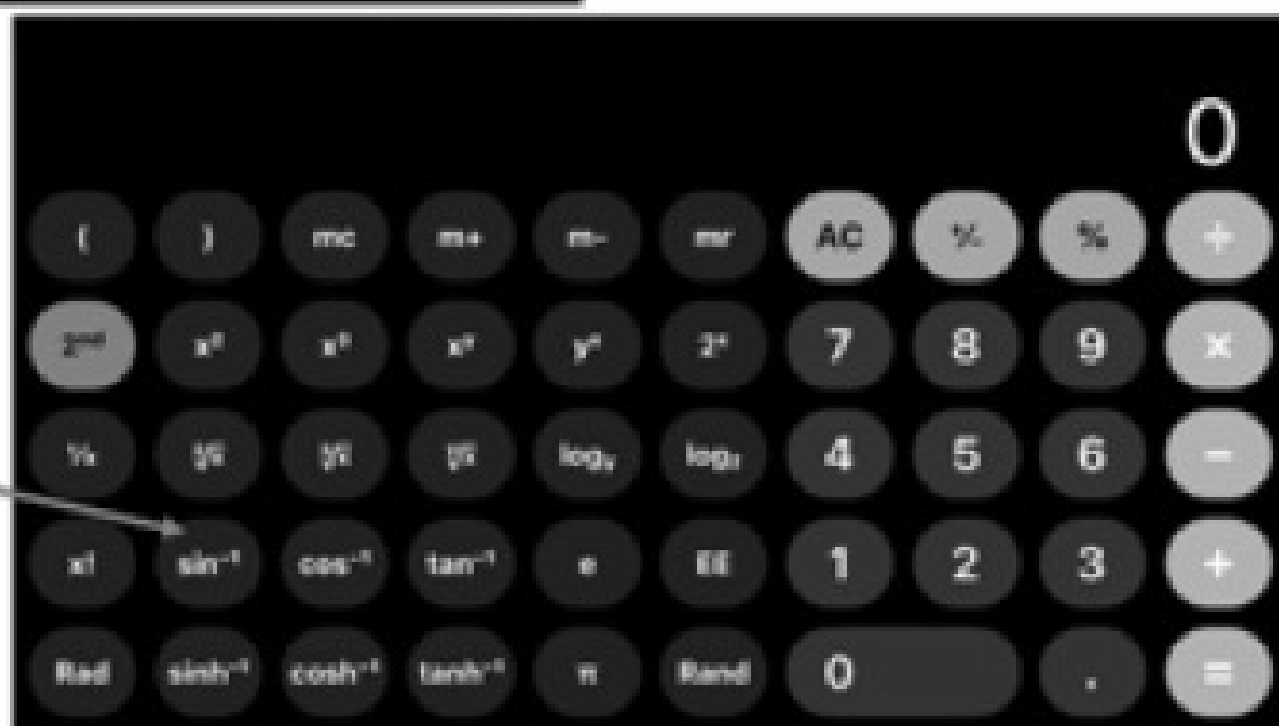
$$\sin = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\text{tangent} = \frac{\text{opposite}}{\text{adjacent}}$$



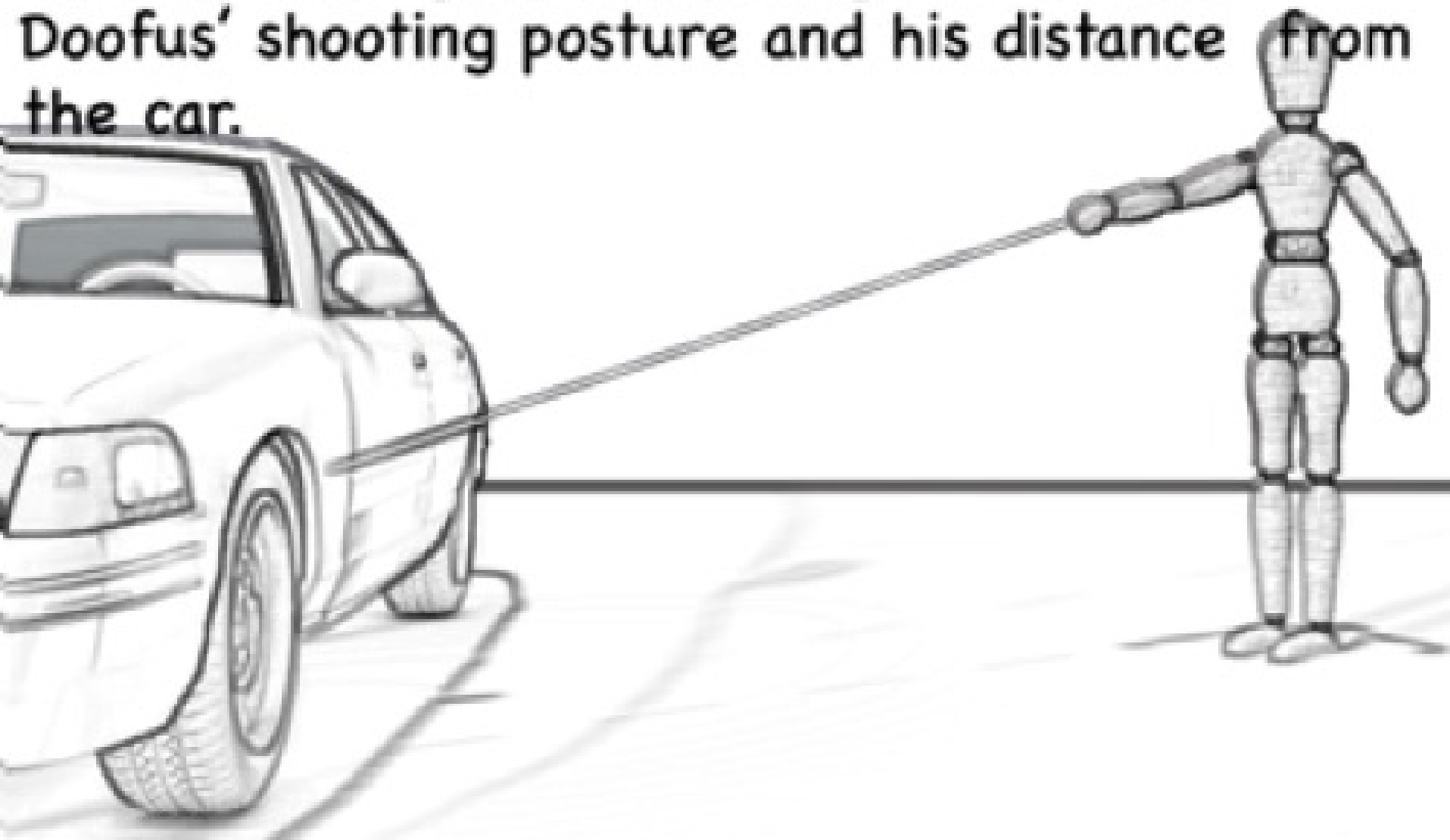
37, sin, — 0.6018

0.6018, sin, — 37



Doofus was observed shooting a car in the parking lot.

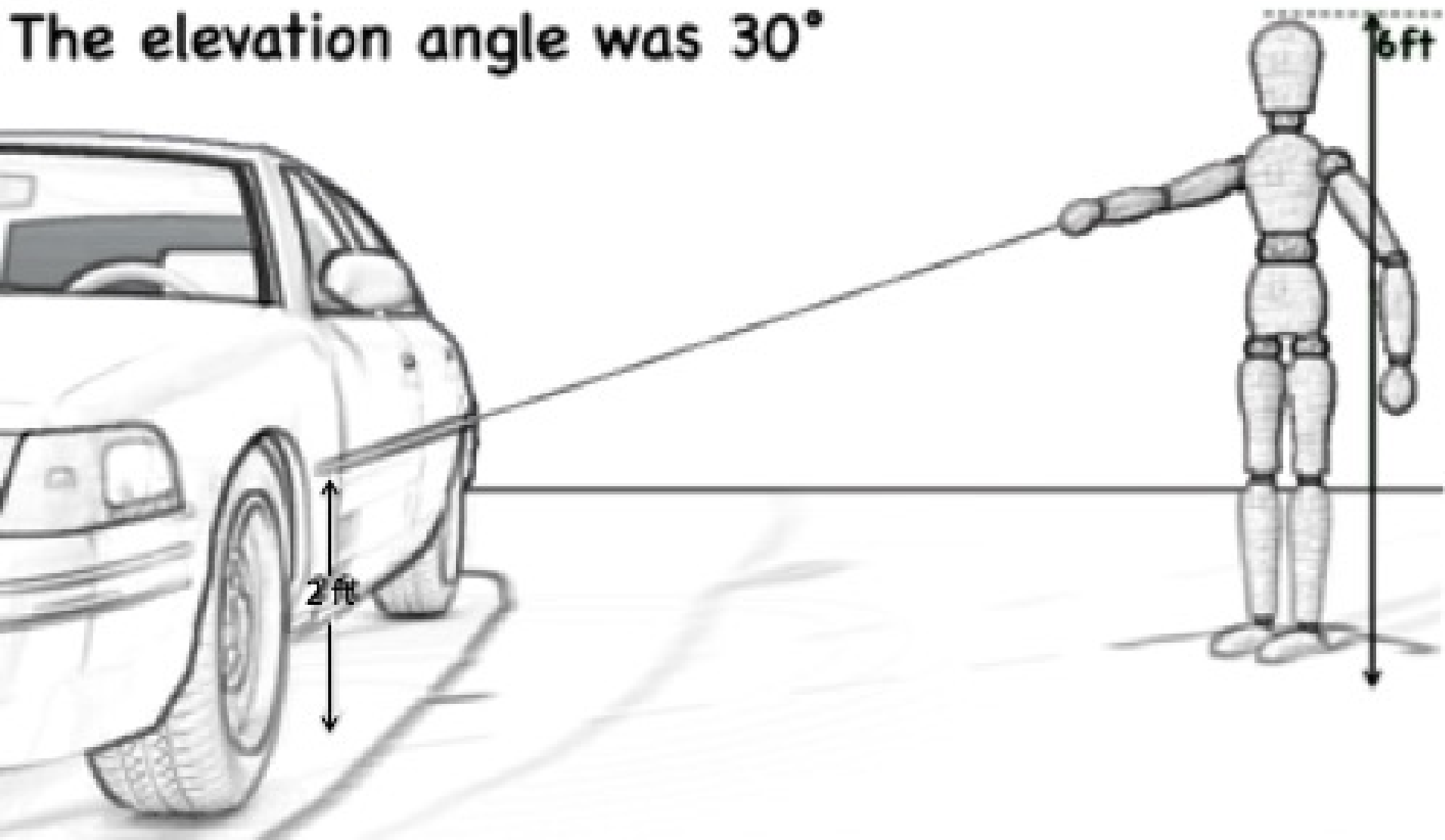
Eyewitnesses gave conflicting accounts of Doofus' shooting posture and his distance from the car.



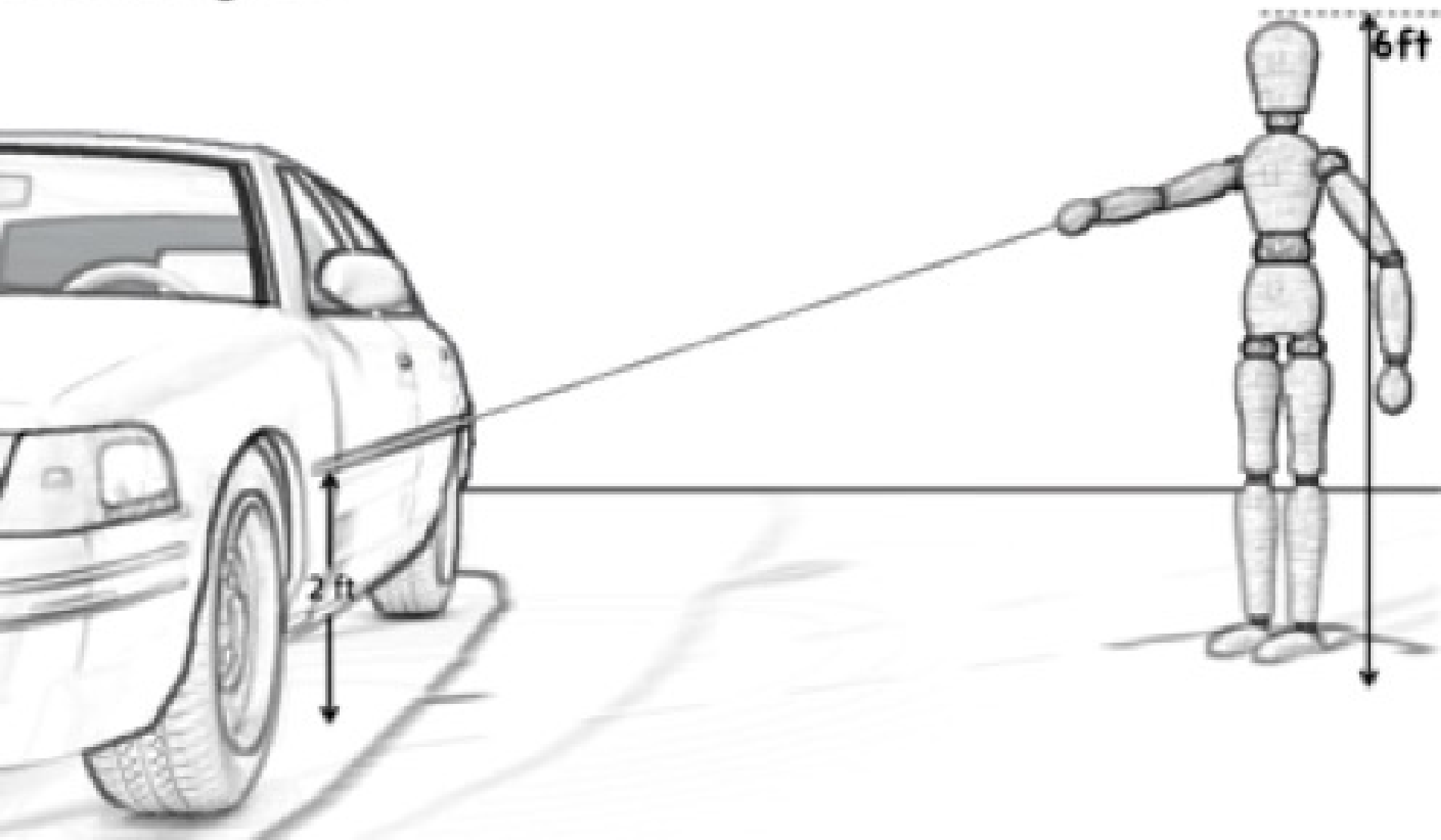
Doofus is 6ft tall.

The height of the bullet defect in the car is 2ft.

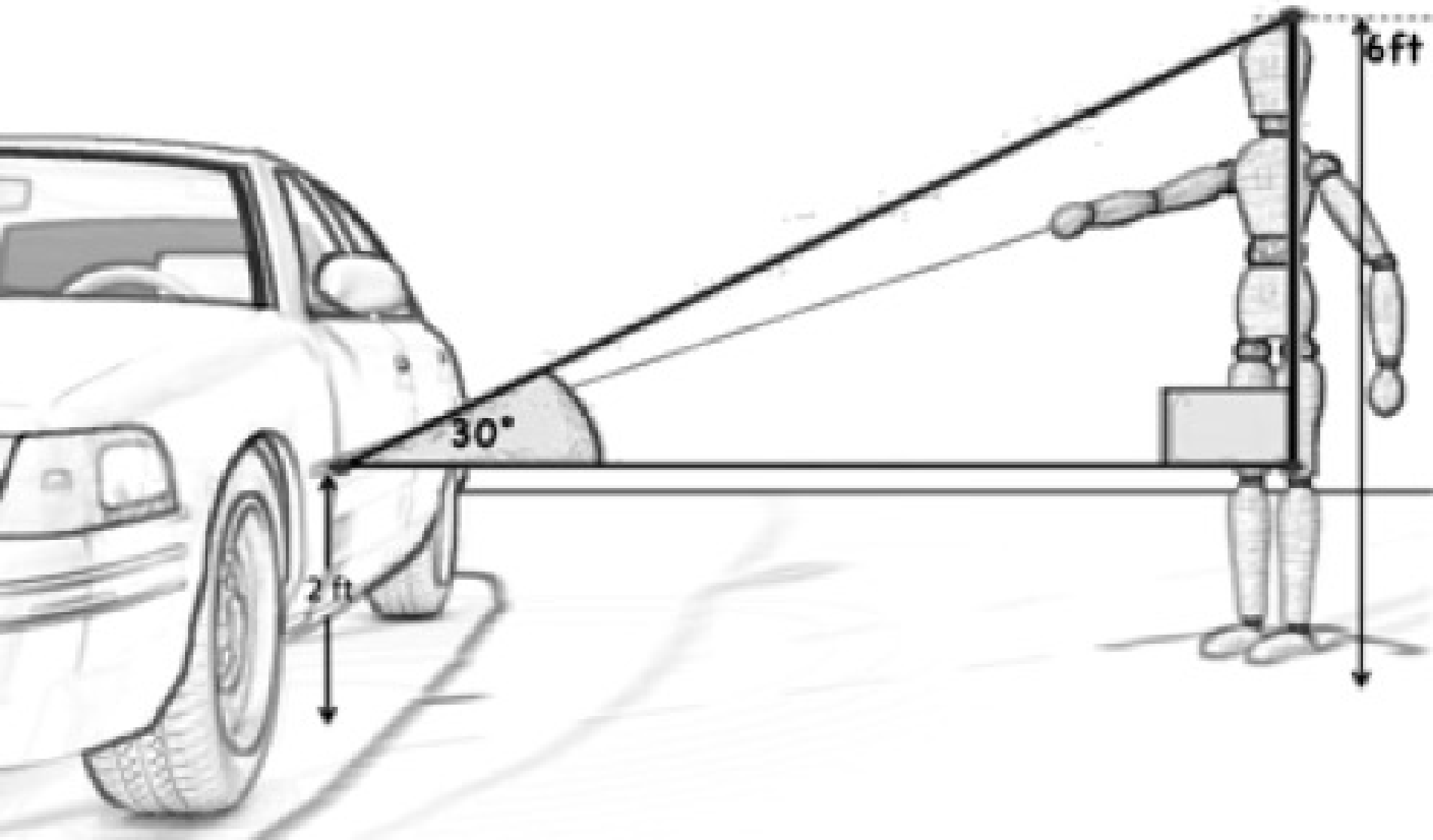
The elevation angle was 30°



Calculate the maximum distance Doofus' firearm was from the car at time the firearm was discharged.



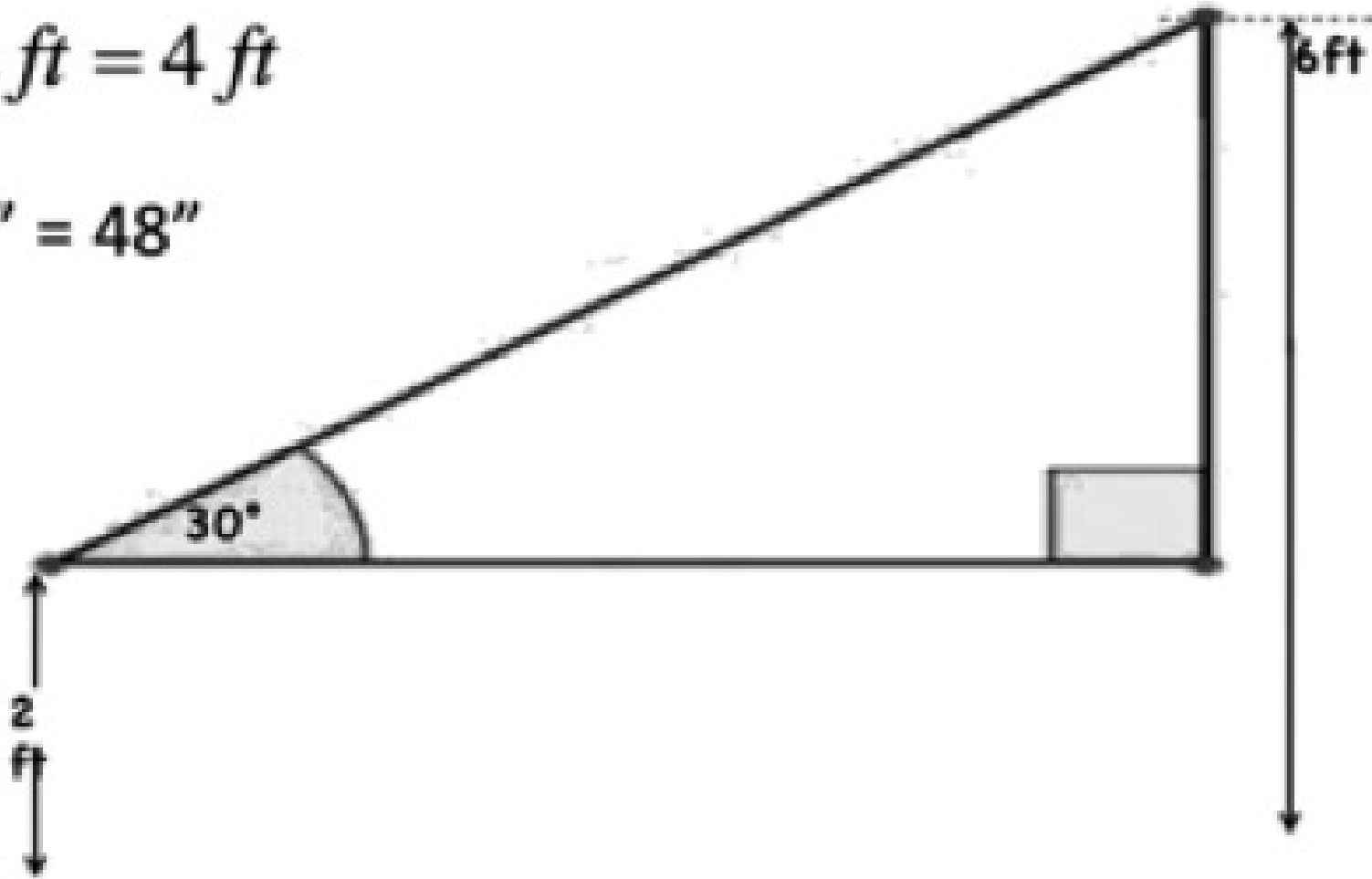
Placing the triangle in the scene image can help you maintain perspective!



What is the height of the opposite side of this triangle?

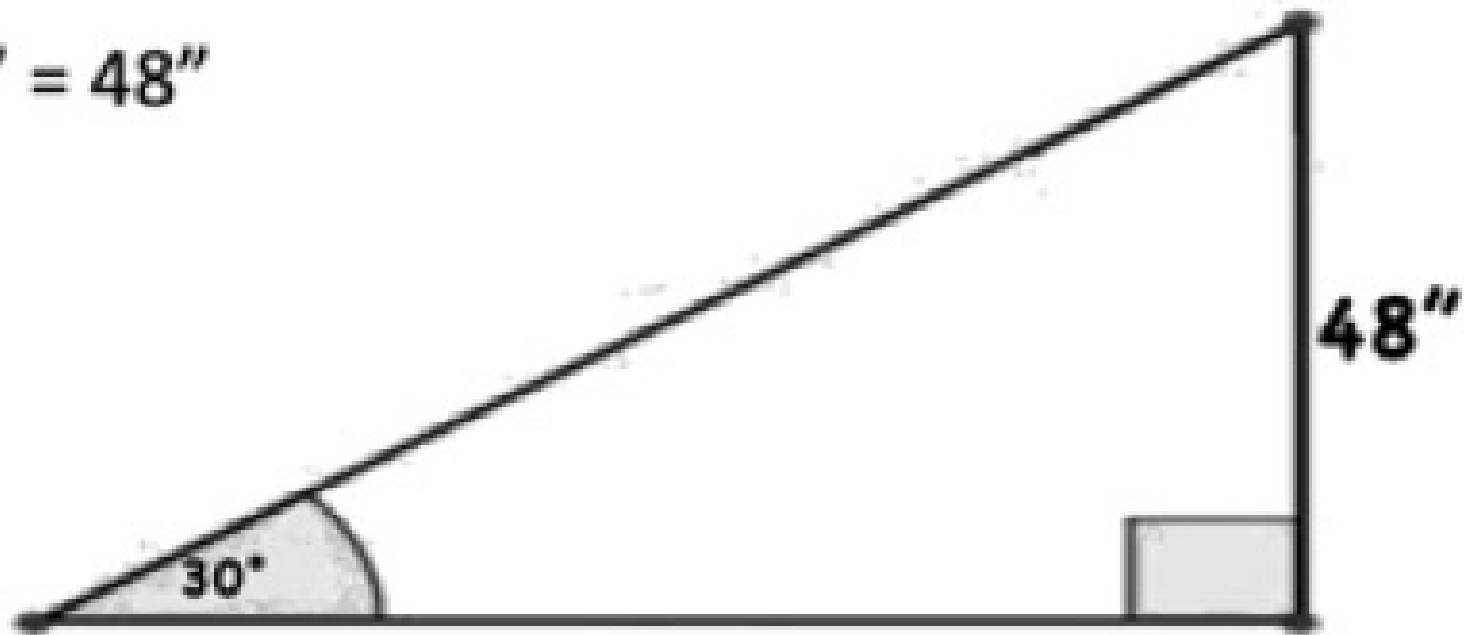
$$6\text{ ft} - 2\text{ ft} = 4\text{ ft}$$

$$72'' - 24'' = 48''$$



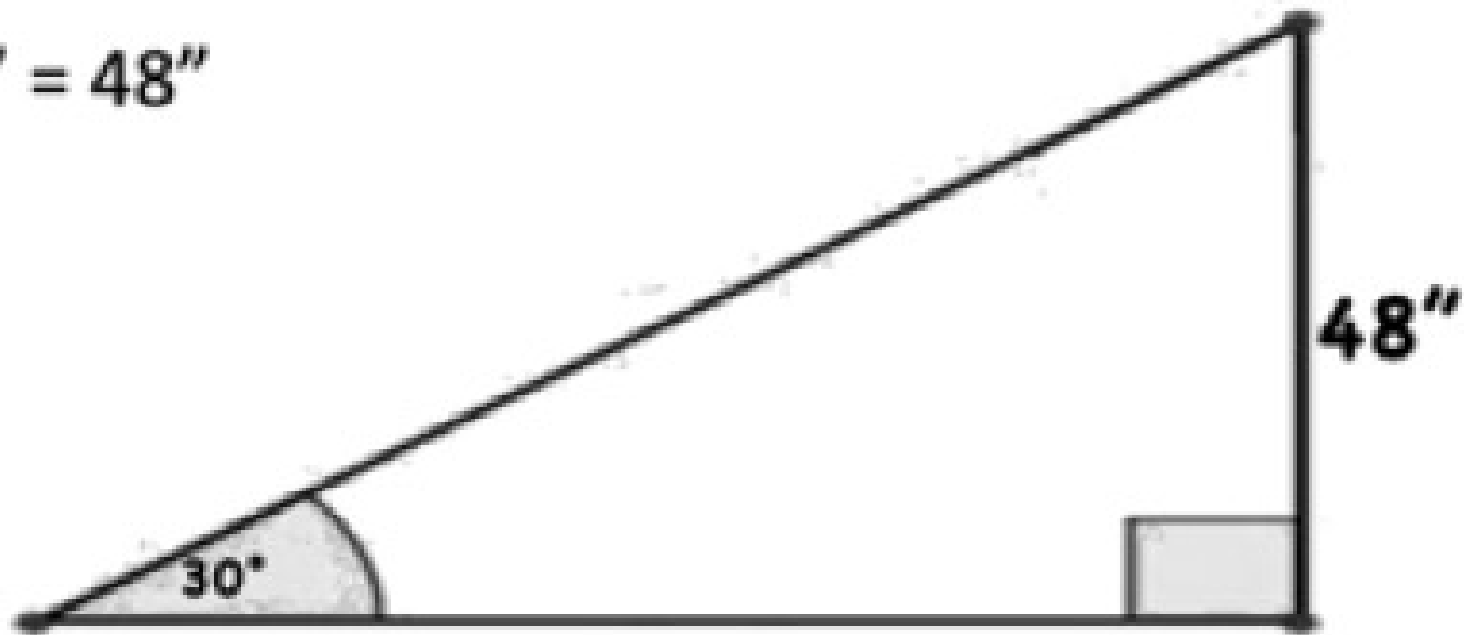
What is the height of the opposite side of this triangle?

$$72'' - 24'' = 48''$$



What is the height of the opposite side of this triangle?

$$72'' - 24'' = 48''$$



“Cheat Sheet” for Tangent Formula

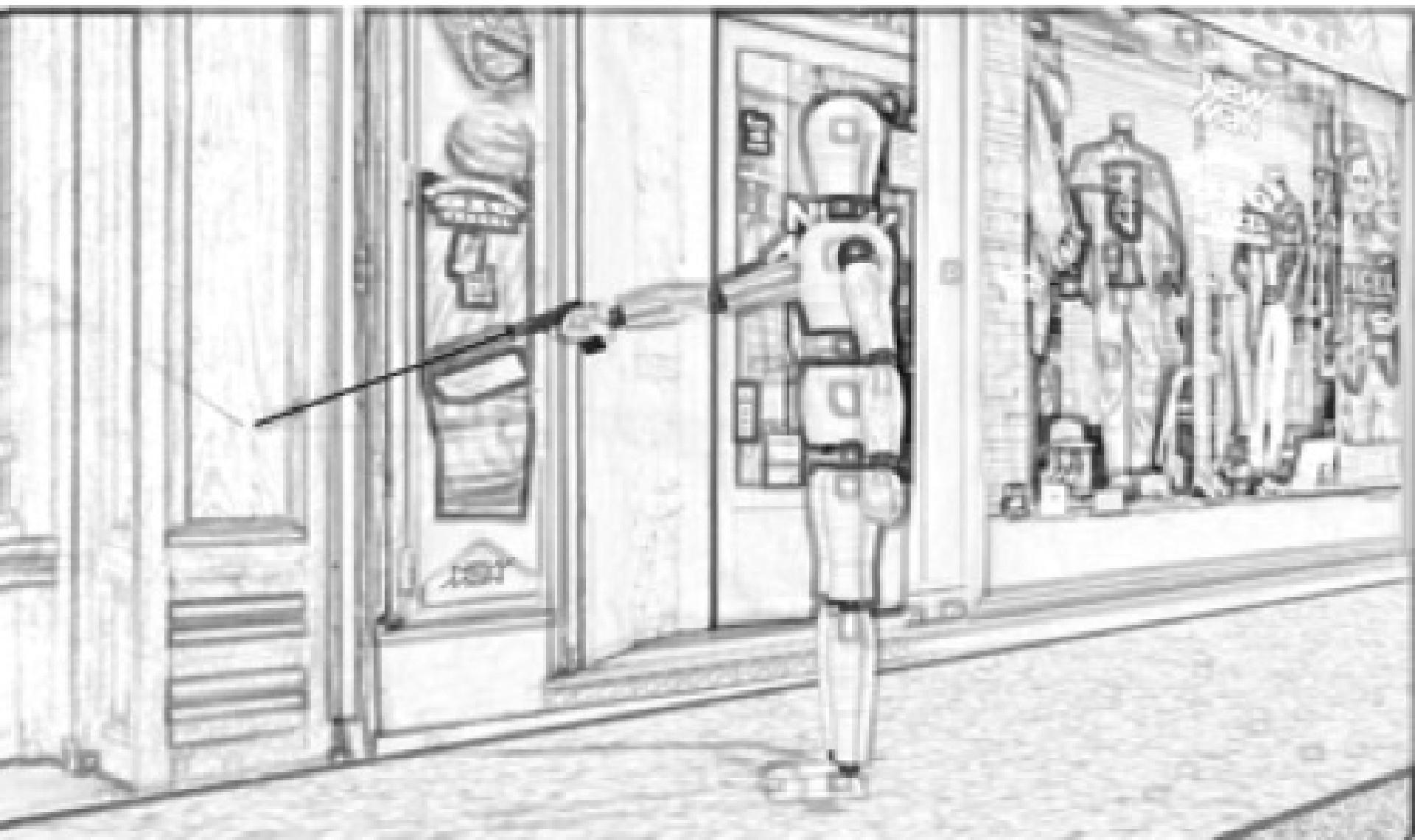
$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\text{opposite} = \tan\theta \times \text{adjacent}$$

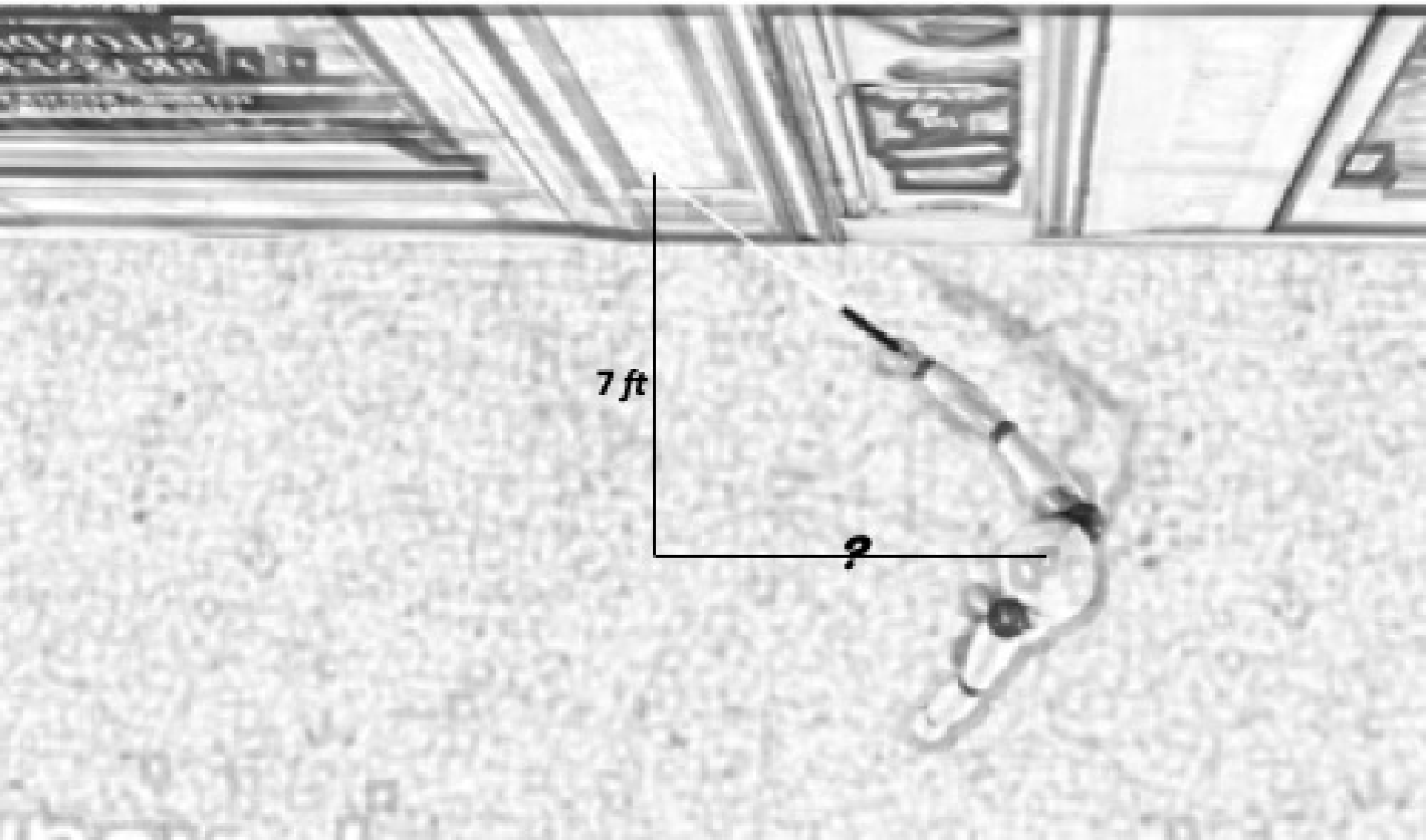


$$\text{adjacent} = \frac{\text{opposite}}{\tan\theta}$$

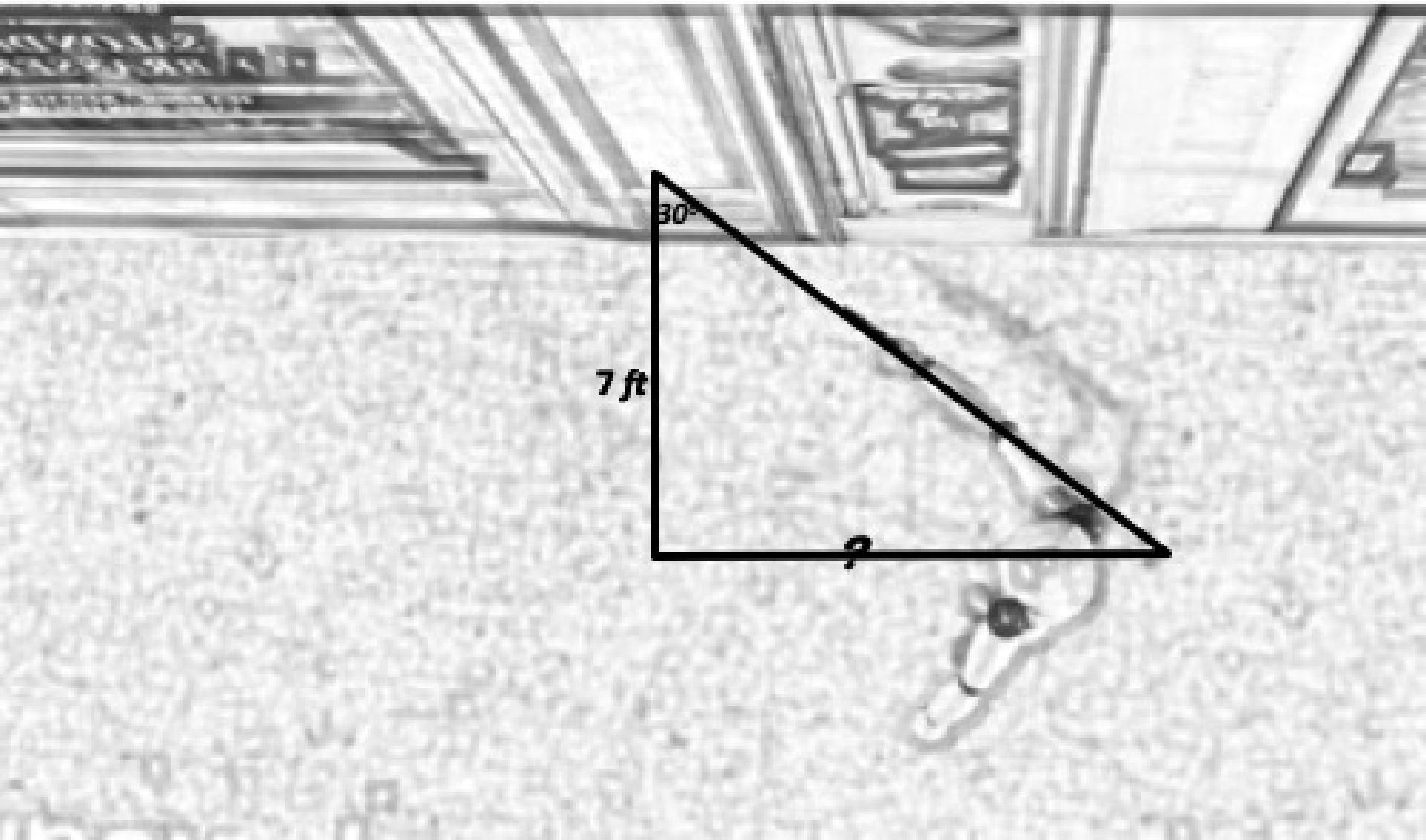
Doofus was observed shooting a storefront.
Eyewitnesses gave conflicting accounts of
Doofus' shooting position and posture.



If Doofus was 7 ft from the wall, but fired at a 30° angle to the wall, (R-L) what was his distance to the right of the defect?



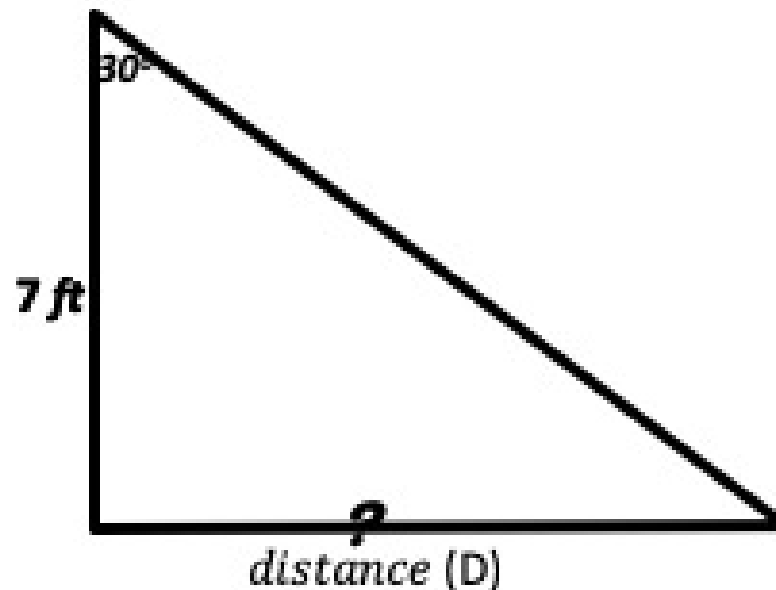
If Doofus was 7 ft from the wall, what was his distance to the right of the defect?



Doofus was 7 ft from the wall. What was his distance to right of the defect?

Opposite=adjacent x tangent 30°

$$\text{distance} = 7 \text{ ft} \times 0.5774 = 4 \text{ ft}$$

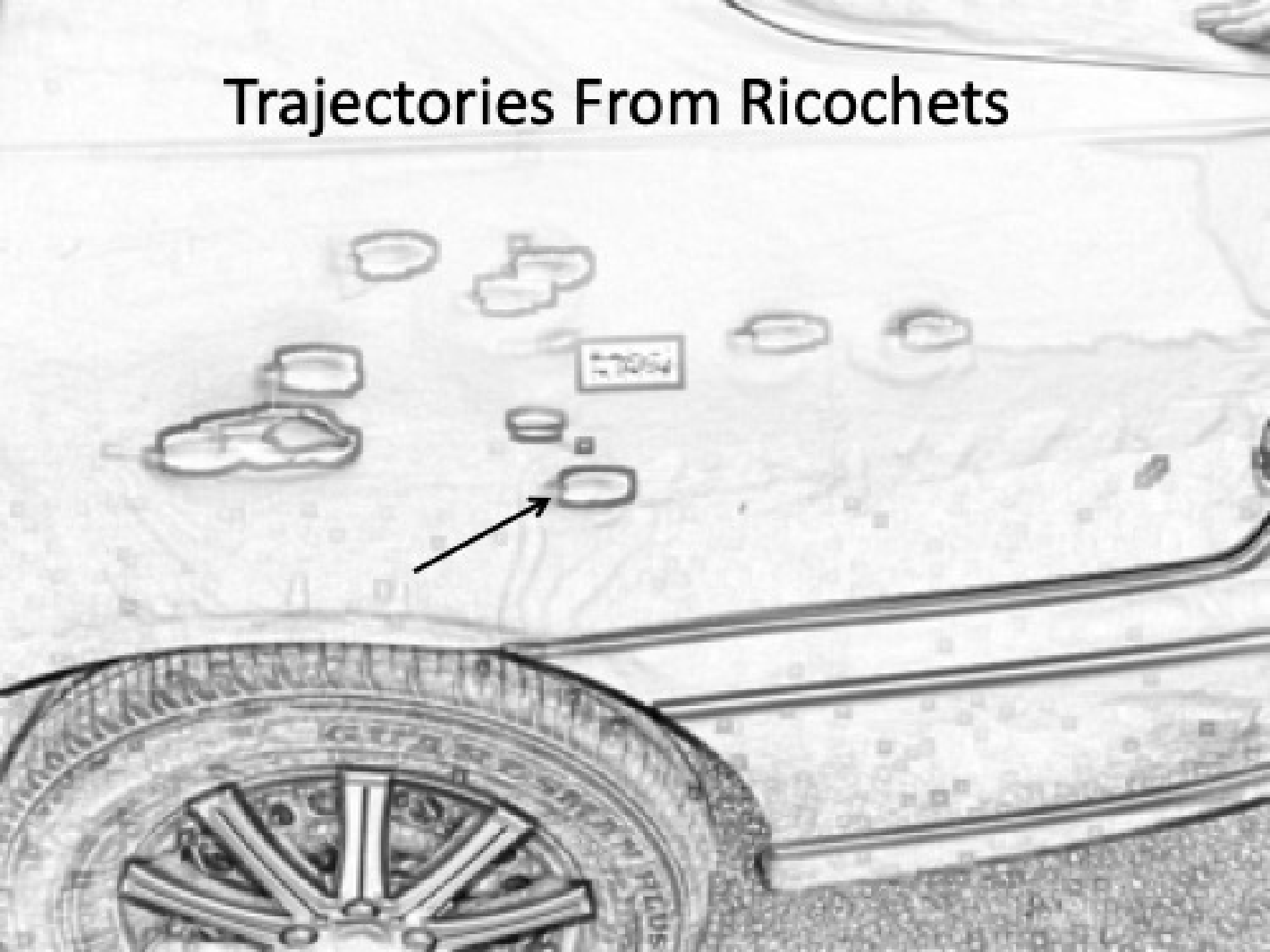


$$\text{tangent} = \frac{\text{opposite}}{\text{adjacent}} \quad \text{adjacent} = \frac{\text{opposite}}{\text{tangent}}$$

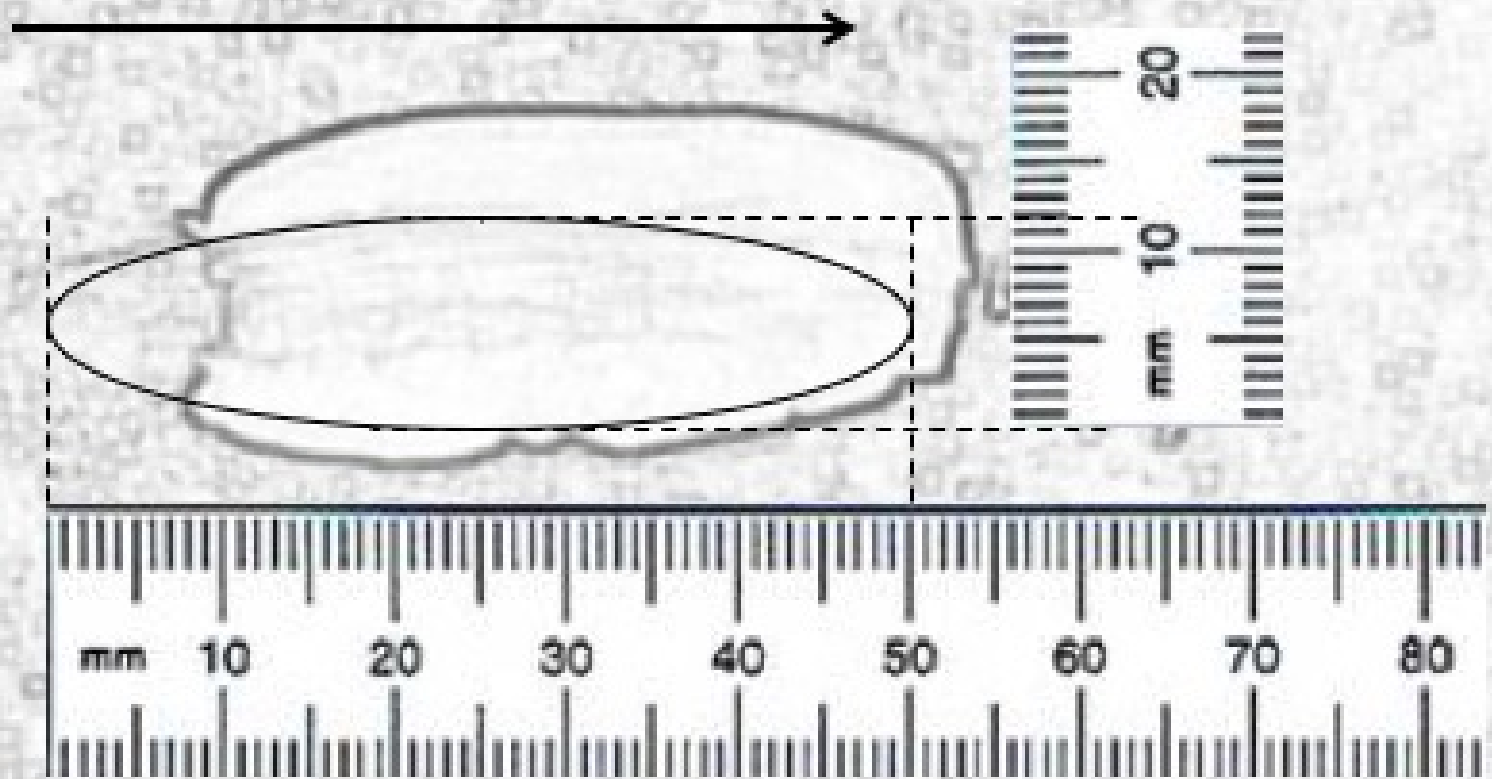
Doing the Math – Tips for Success

- Convert all data to inches
 - Work through problem, then convert answer.
- Round fractions to four decimal places
 - $\tan 30^\circ = 0.5774$
 - Include the 0 in decimals.... 0.3476
- In most cases, round degrees to closest degree.
- Visualization – Draw pictures.
- Audit work. Mistakes begat mistakes.

Trajectories From Ricochets



Incident Angle from Sin Method



$$\frac{\text{width}}{\text{length}} = \sin \theta$$

$$\frac{12 \text{ mm}}{50 \text{ mm}} = 0.24$$

14 degrees

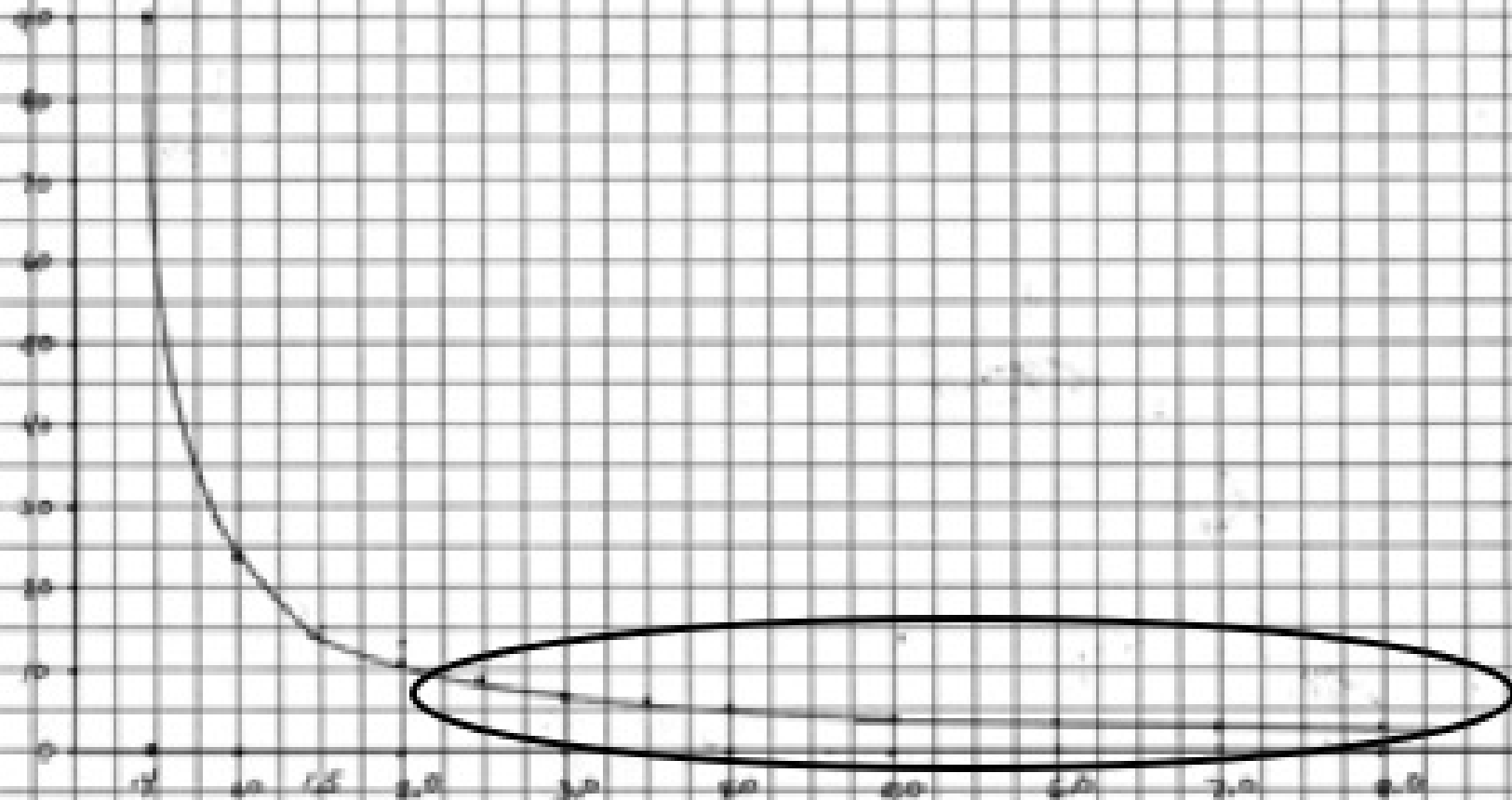
BULLET DEFECT ANALYSIS

WIDTH = 0.4" = CONSTANT

$$\sin \theta = \frac{W}{L}$$

$$L = \frac{W}{\sin \theta}$$

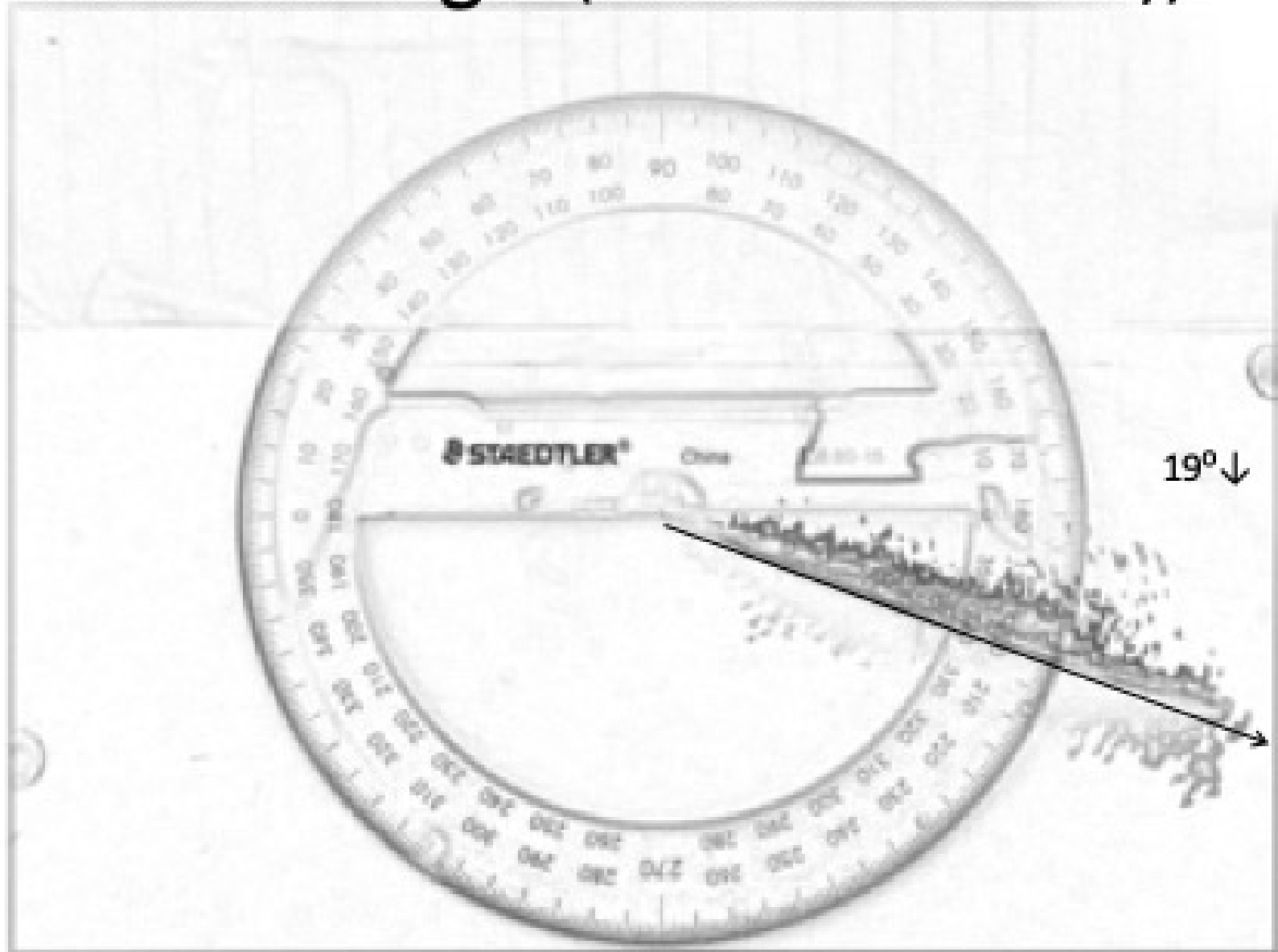
INCIDENT
ANGLE



LENGTH INCHES

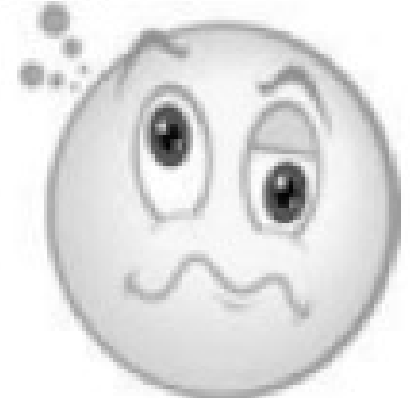
1" = 33.6°	3" = 7.3°	6" = 3.6°
1.5" = 18.5°	3.5" = 6.6°	7" = 3.3°
2" = 14.5°	4" = 5.7°	8" = 2.9°
2.5" = 11.2°	5" = 4.6°	

Elevation Angle (Vertical surfaces only)



Bullet Strike on Intermediate objects

- Ricochet: Continued flight of a rebounded projectile after a low-angle impact with a surface or object.
...deflection without penetration or perforation.
 - (Haag 2nd edition p 144)
- Trajectory continues in same plane.
- Deflection: *as result of a ricochet*: departure trajectory in different plane.
 - Lateral deviation



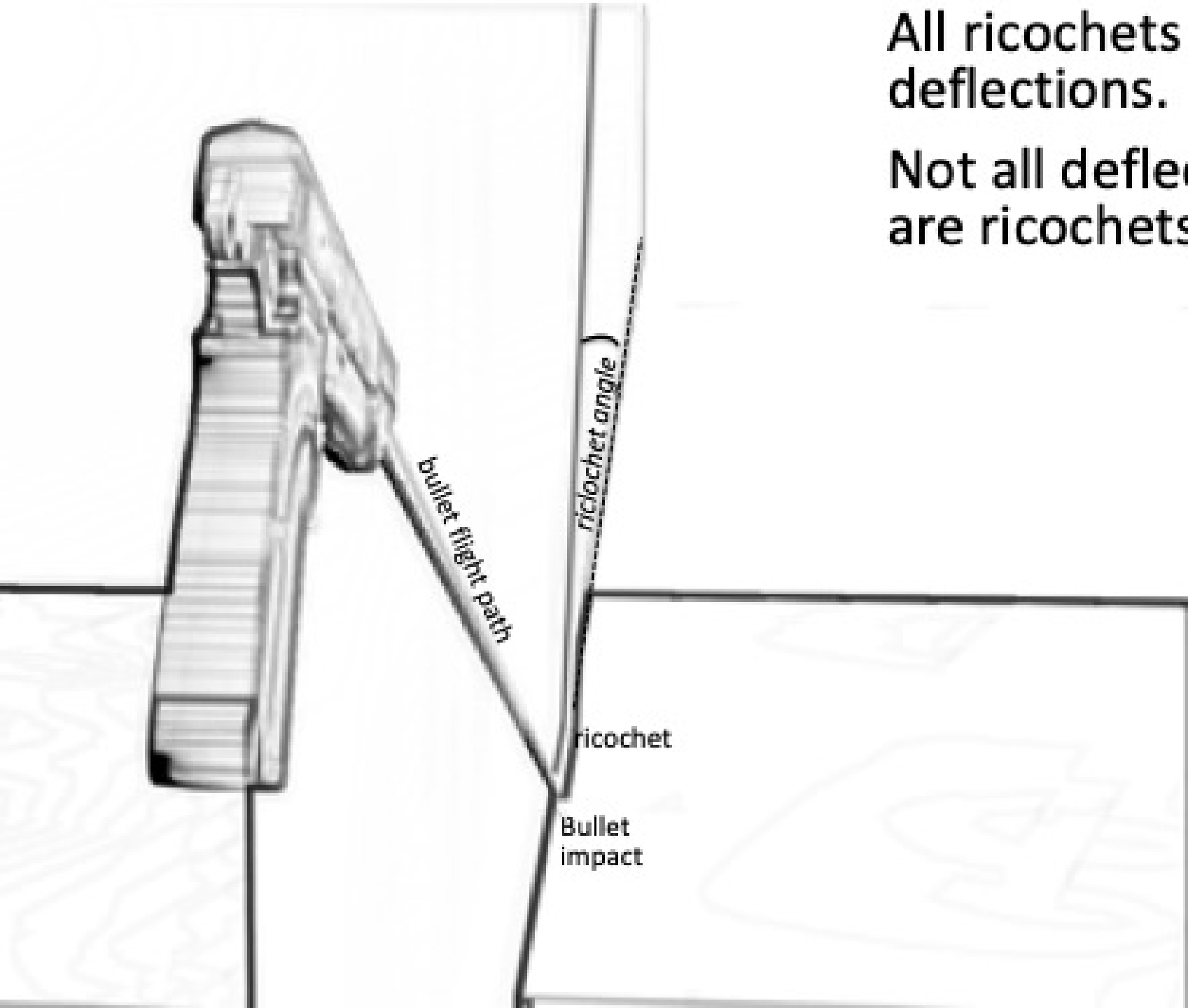
Bullet Strike on Intermediate objects

- Deflection: *as result of perforating or striking an object*: Deviation in any direction from normal flight path.
- Deflection: *as result of a ricochet*: refers to the horizontal or azimuth component of the angle of departure.



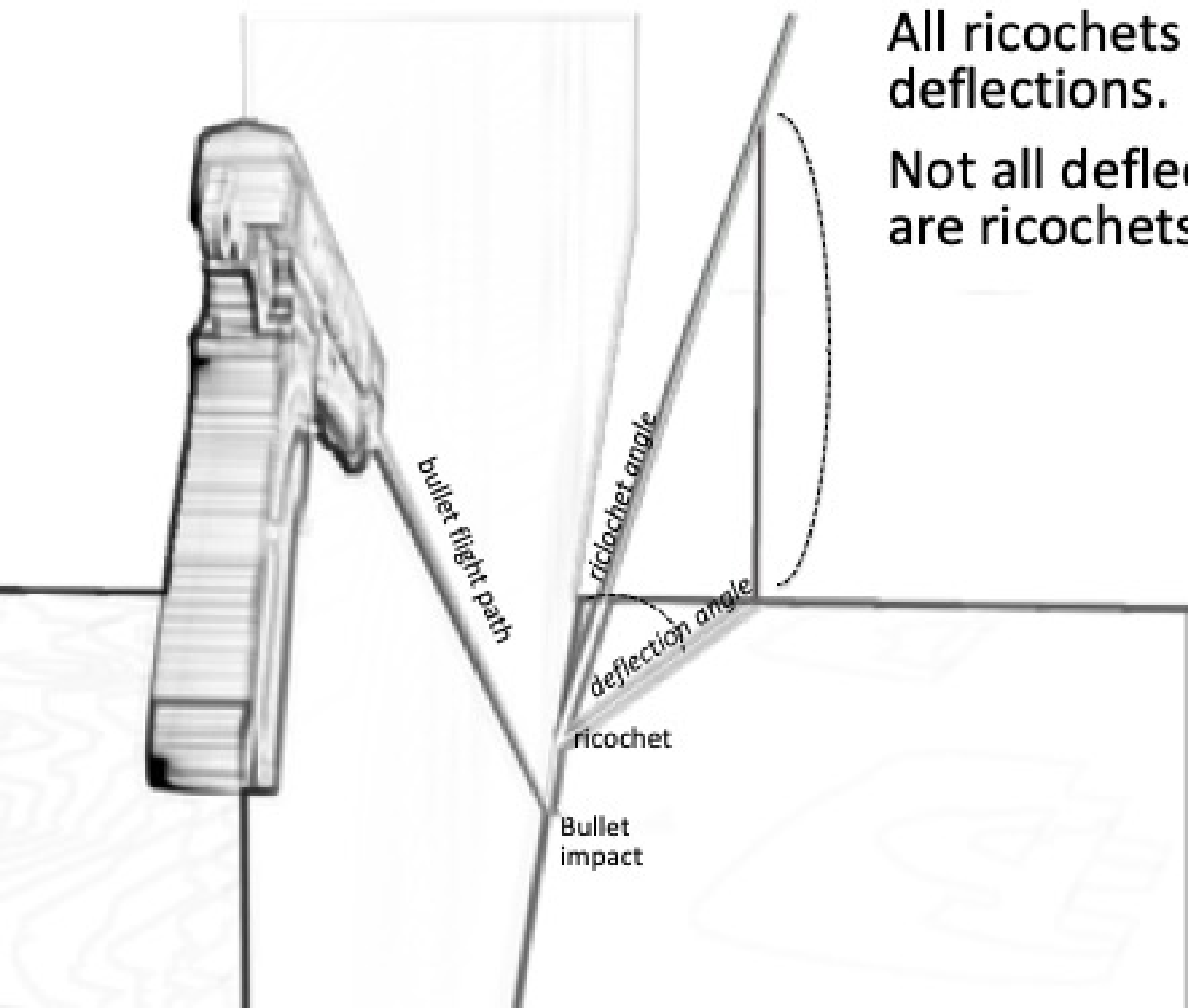
All ricochets are deflections.

Not all deflections are ricochets.



All ricochets are deflections.

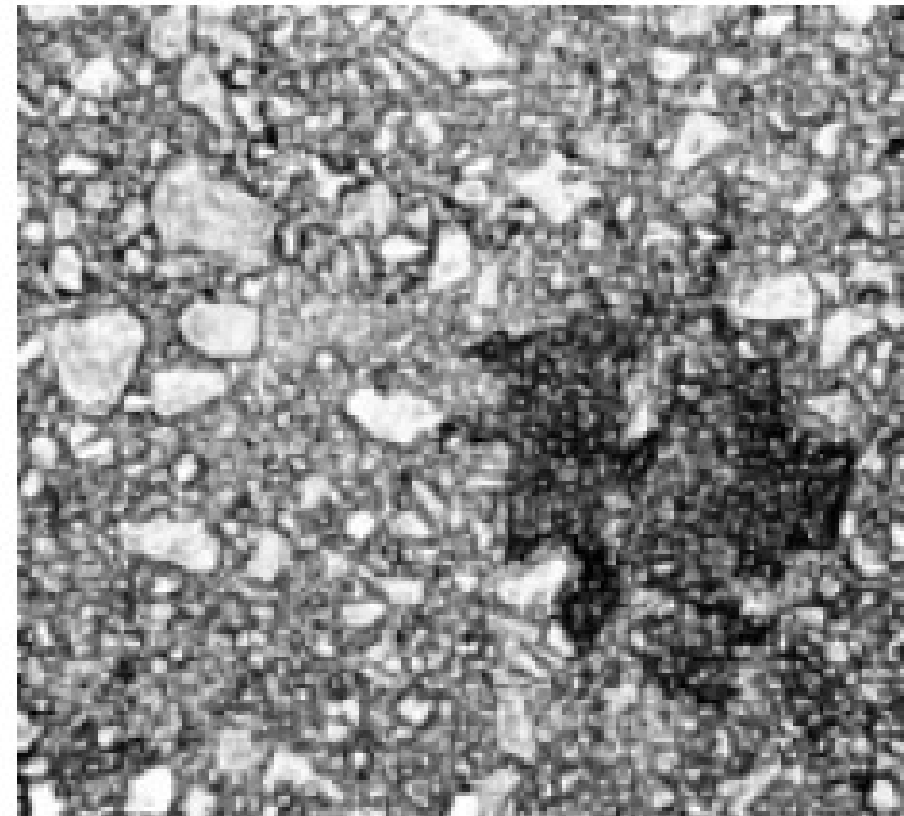
Not all deflections are ricochets.

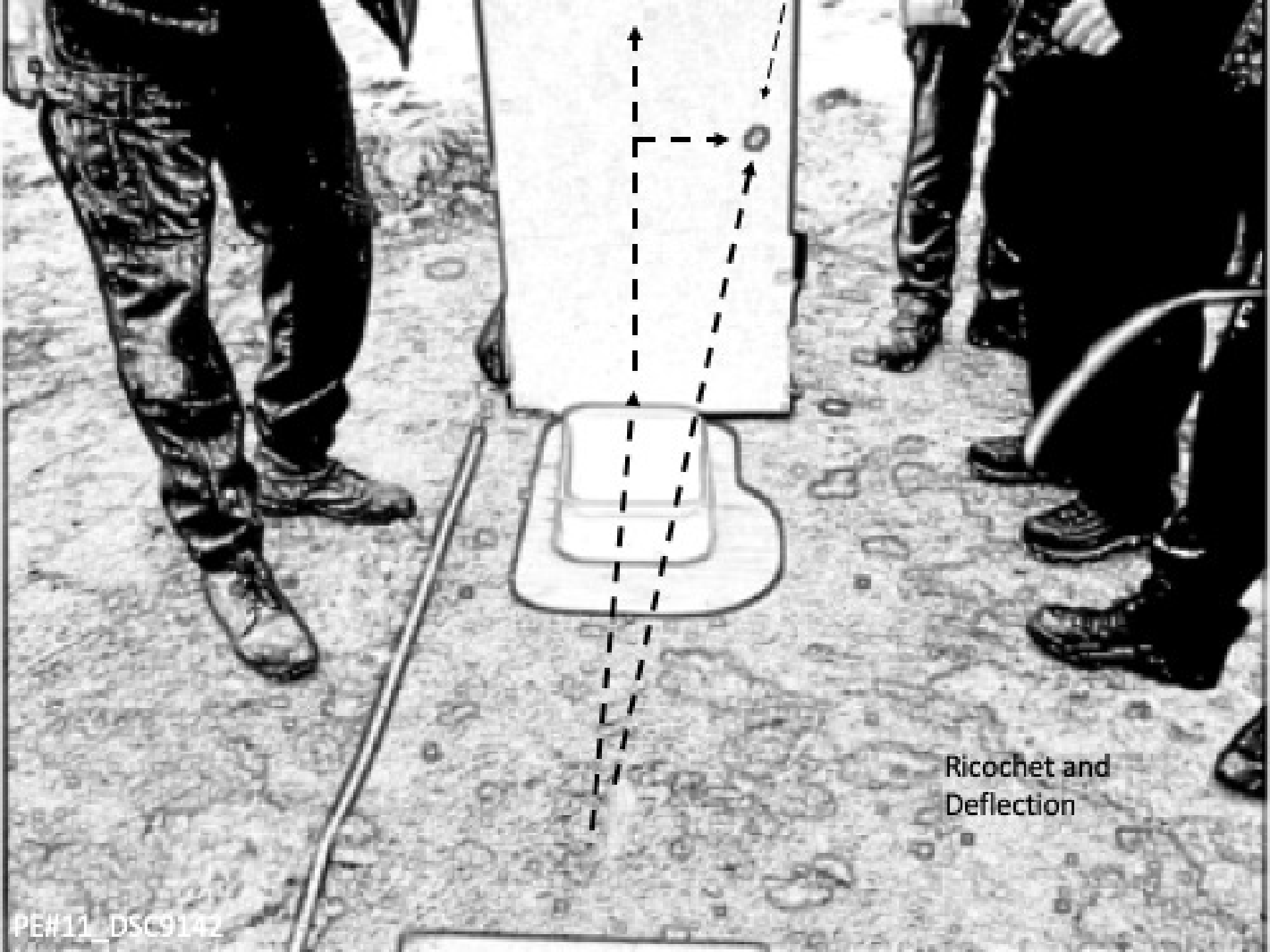


Deflections negligible from ricochets off hard smooth surfaces. (steel, tile, concrete)

Deflections more likely from ricochets off uneven surfaces such as asphalt, sand, dirt.

Asphalt
spawl

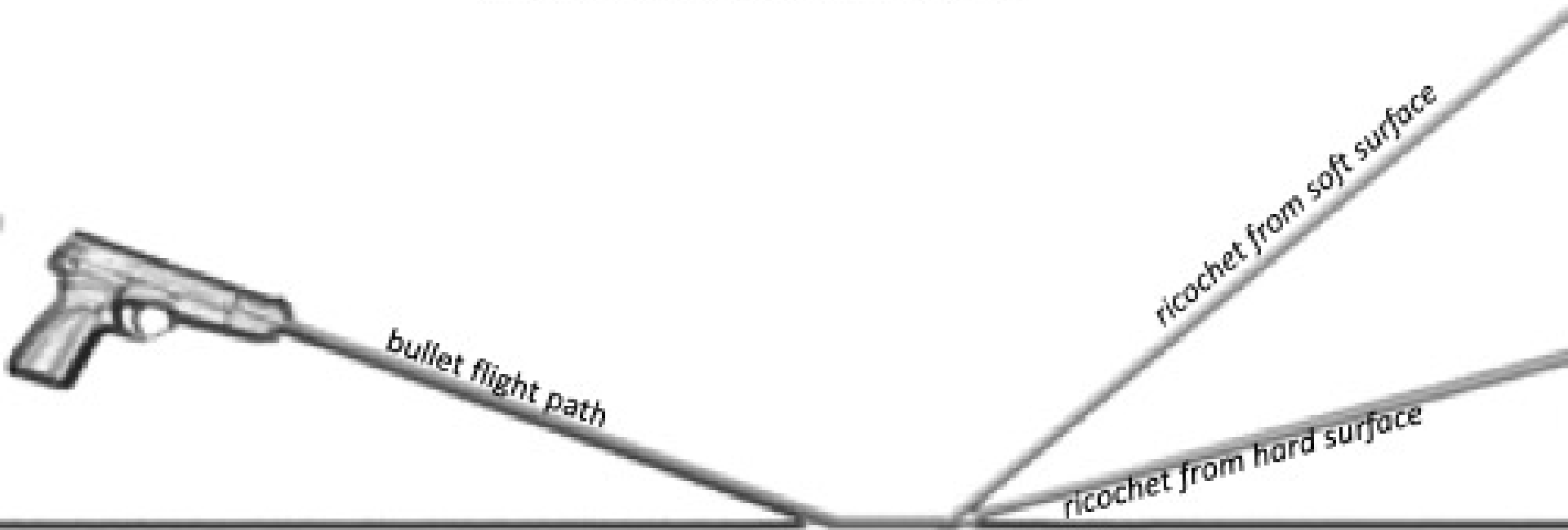




Ricochet and
Deflection

Ricochets

Hard surface – Expect lower ricochet angle (steel, concrete)
Softer surface – May have higher ricochet angle (sand, gravel, asphalt)



Ricochets

- Potential evidence at scene from ricochet:
 - Ricochet impact site
 - Directionality, incident angle determination

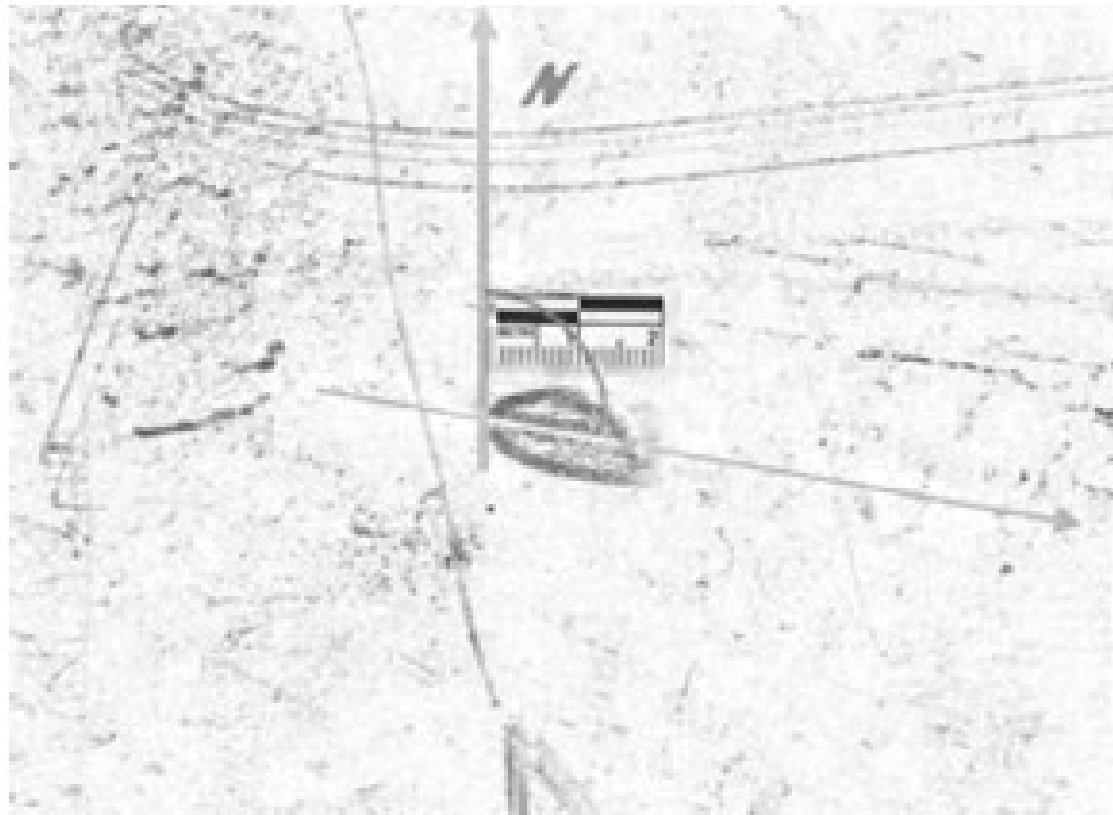
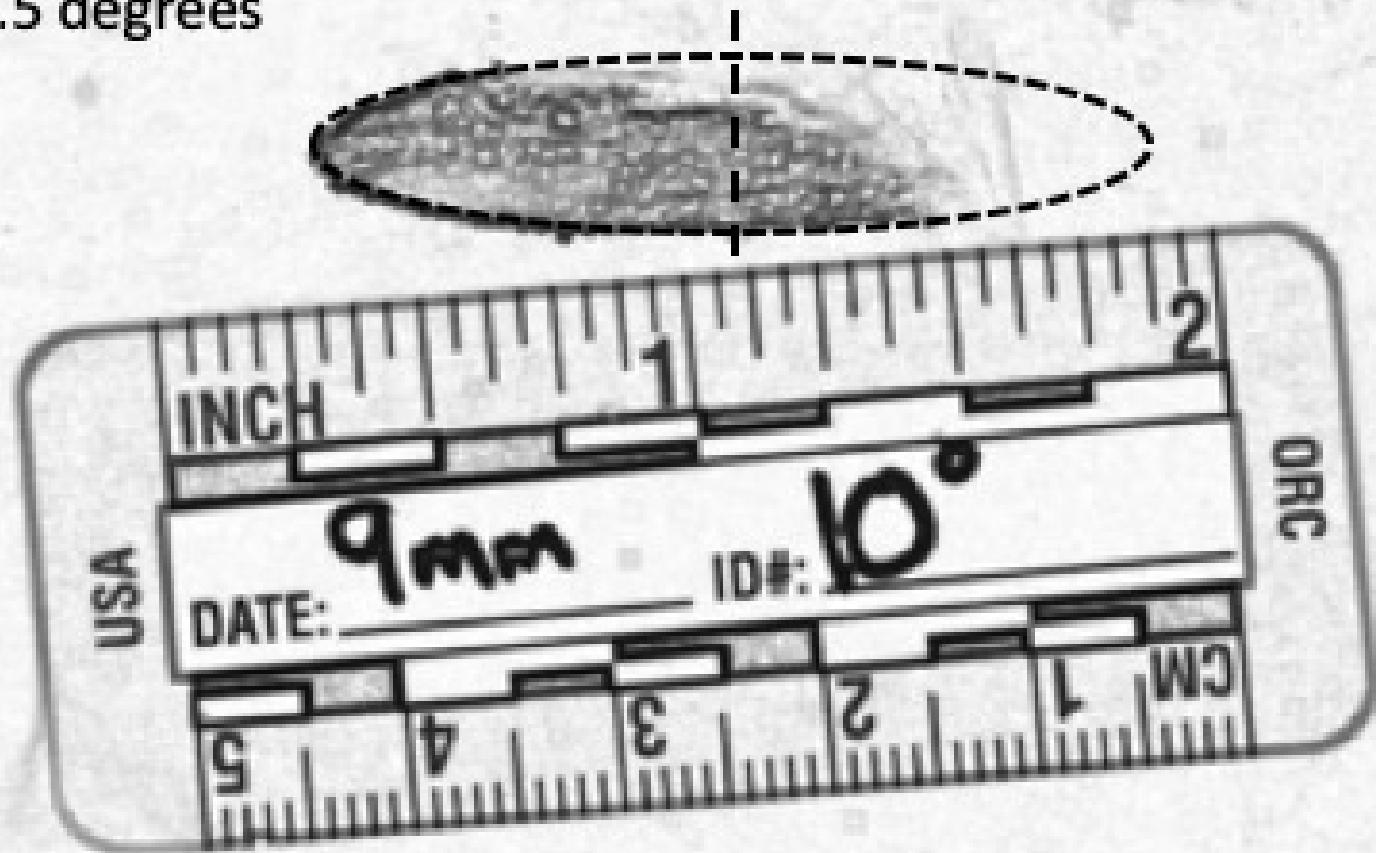


Figure 1 BDF#1 Gamma Angle= 101°
(11° from East-West reference)

Ricochet off Plate Steel, Horizontal Surface

$\sin \text{ Incident Angle} = W/L = 8/40 = 0.2000$

$A = 11.5 \text{ degrees}$



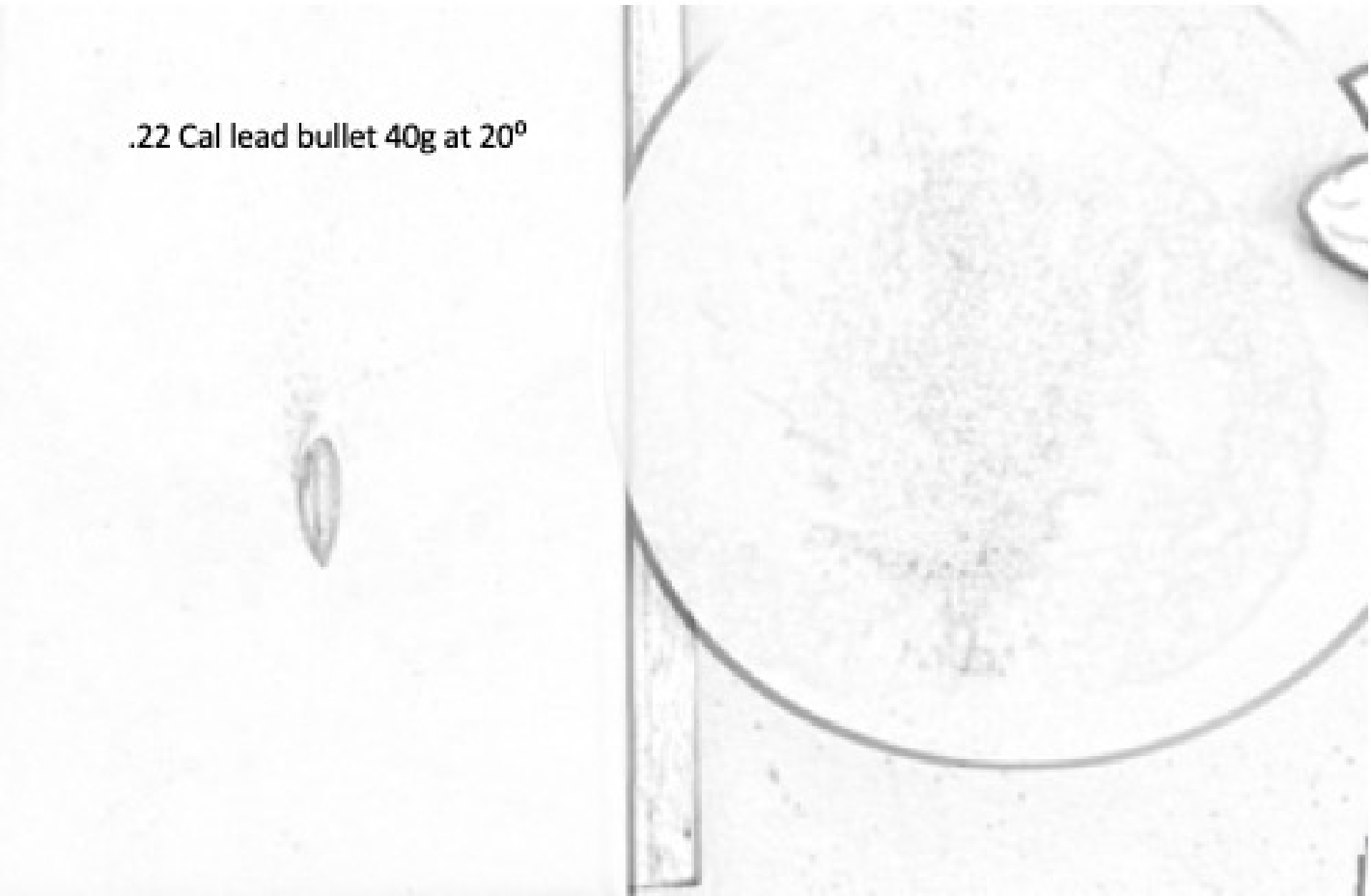
Partial Ellipse: Estimate half and double

Ricochets

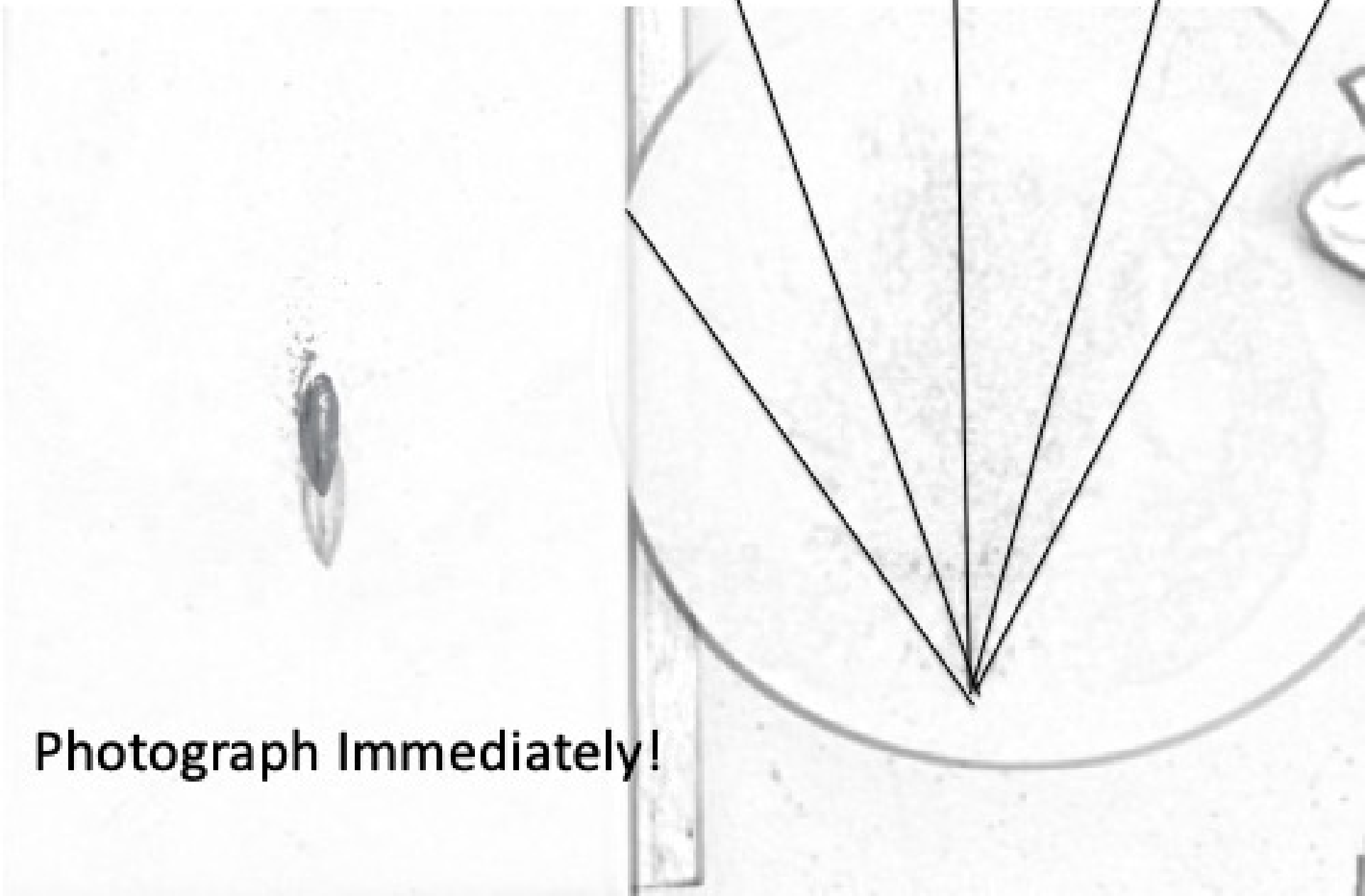
- Potential evidence at scene from ricochet:
 - Ricochet impact site
 - Directionality, incident angle determination
 - Trace evidence
 - Lead splash test

Lead Splash Test

.22 Cal lead bullet 40g at 20°



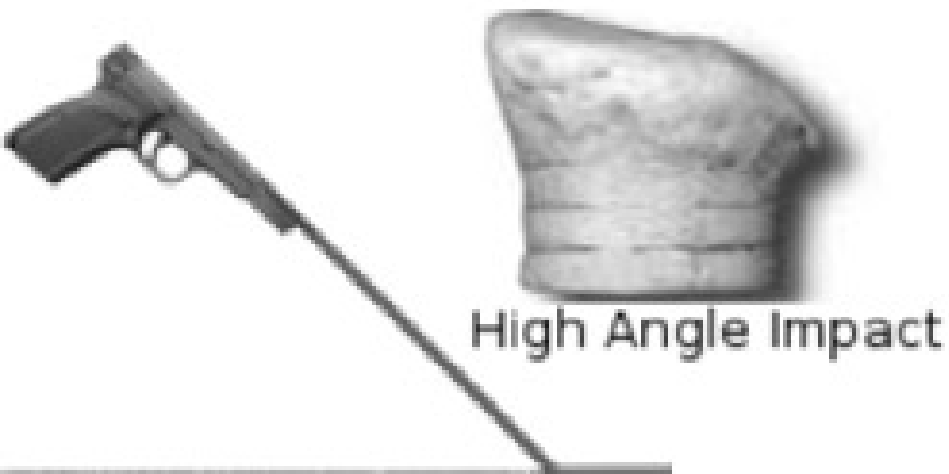
Lead Splash Test - Directionality



Photograph Immediately!

Ricochets

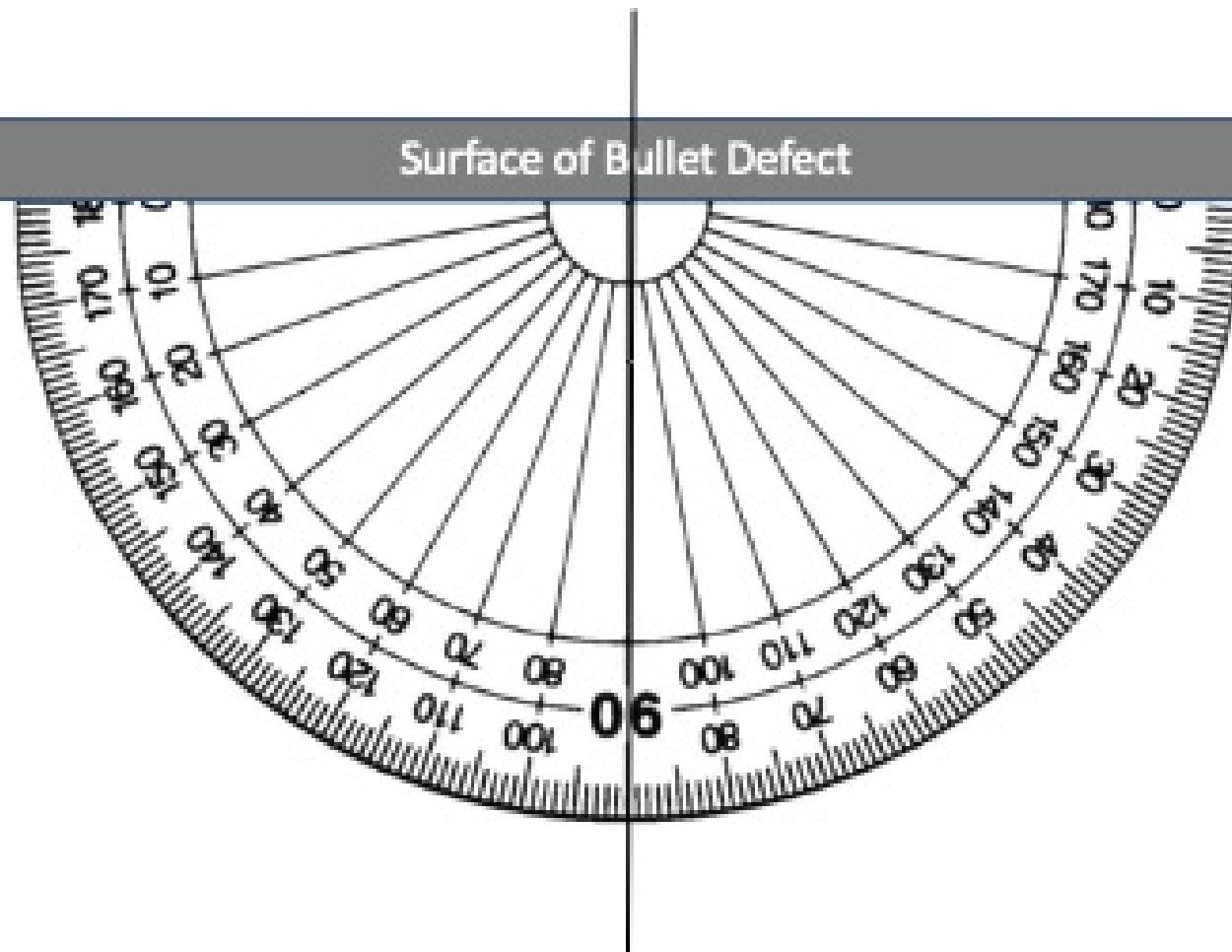
- Potential evidence at scene from ricochet:
 - Ricochet impact site
 - Directionality, incident angle determination
 - Trace evidence
 - Lead splash test
 - Deformed bullet



Ricochets – Tips for Success

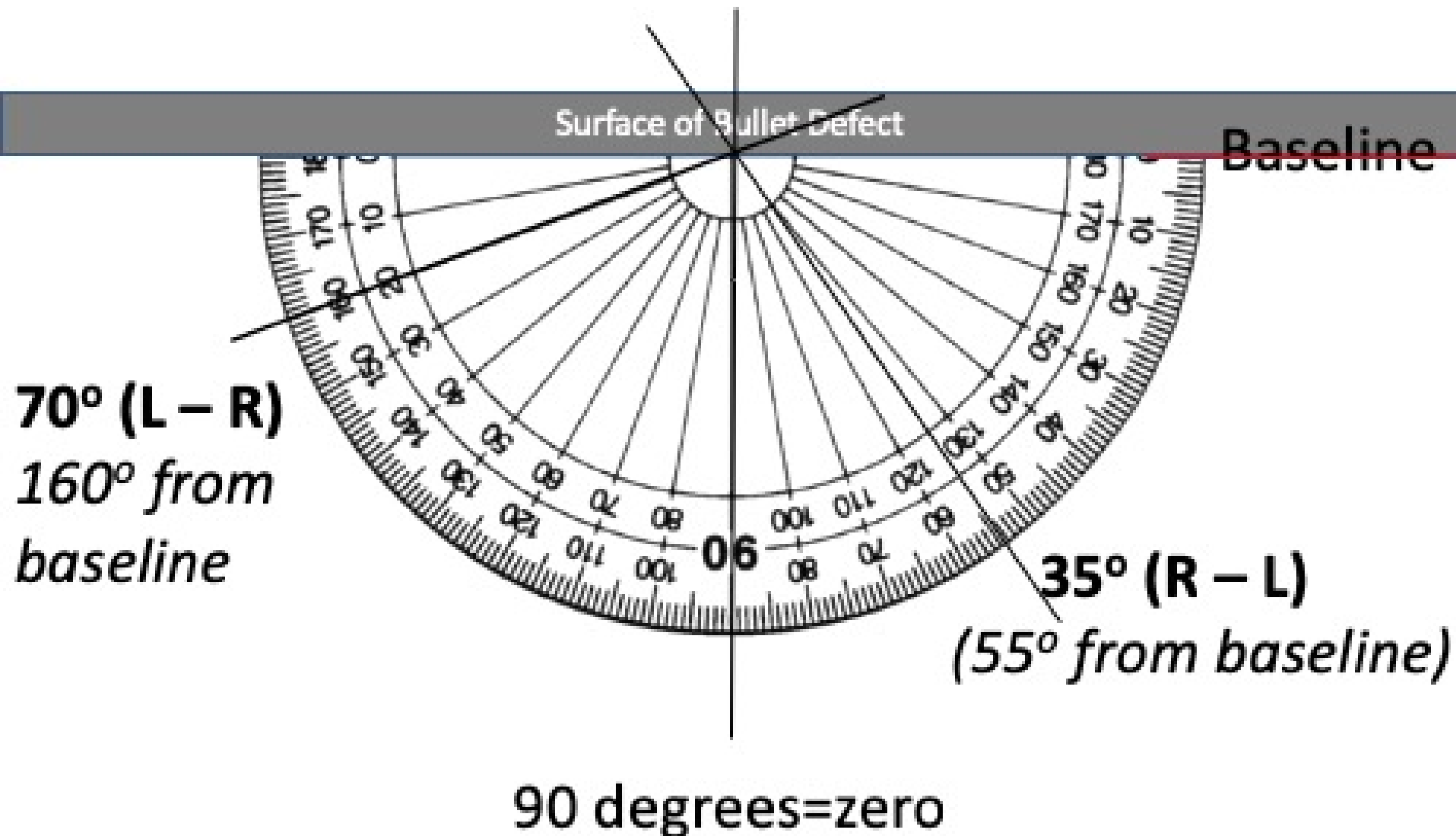
- Shallow angles, less than 10°
- Proper documentation
 - Orthogonal views – scale in plane
 - Gamma angle noted
- Identifying the ellipse
 - Total length and/or $\frac{1}{2}$ and double

Reading Azimuth Angles (Top View)



90 degrees=zero

Reading Azimuth Angles (Top View)



VERTICAL ANGLE



plumb

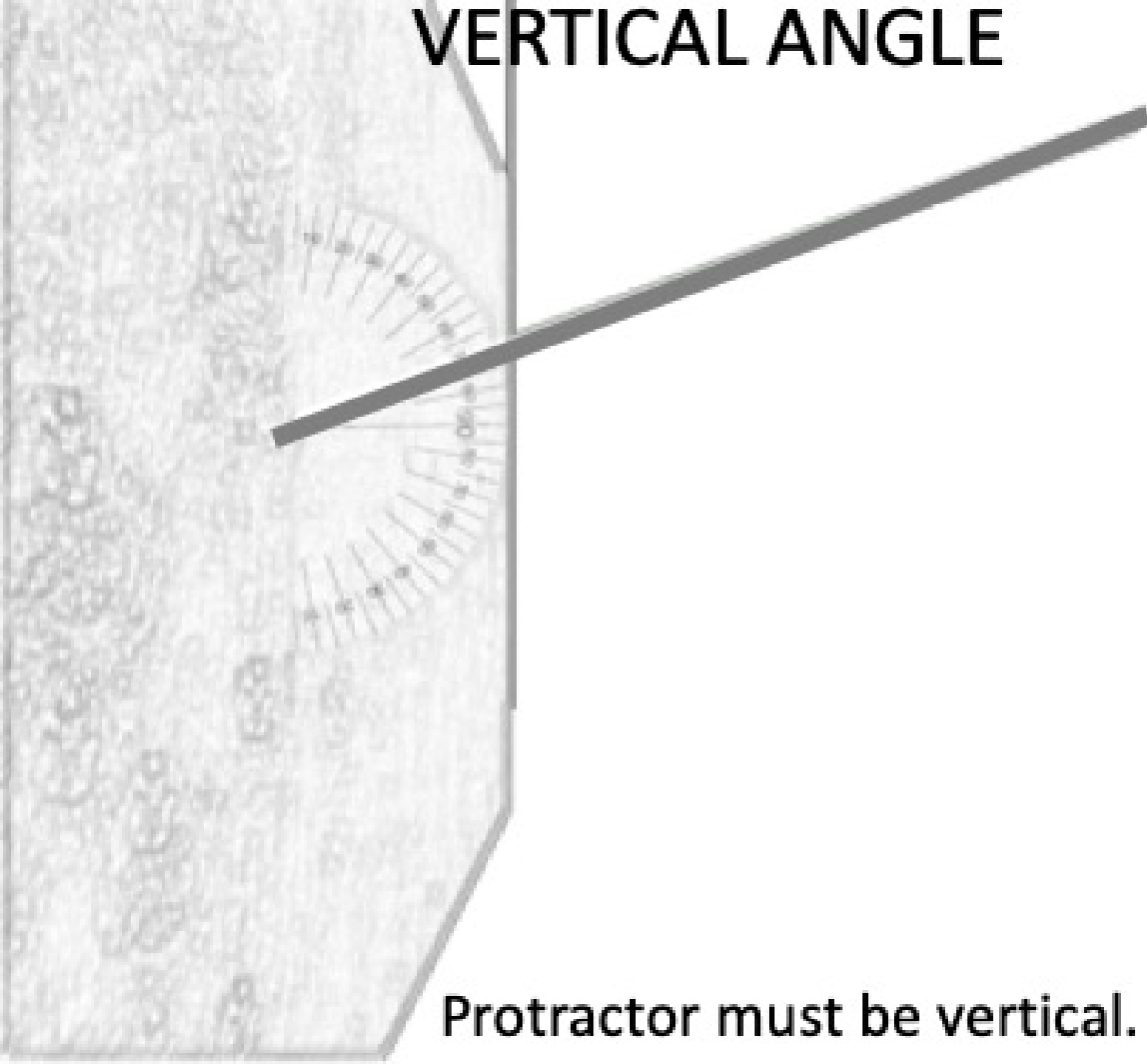
rod

horizontal

90°

- DEGREES UP OR DOWN FROM HORIZONTAL

VERTICAL ANGLE



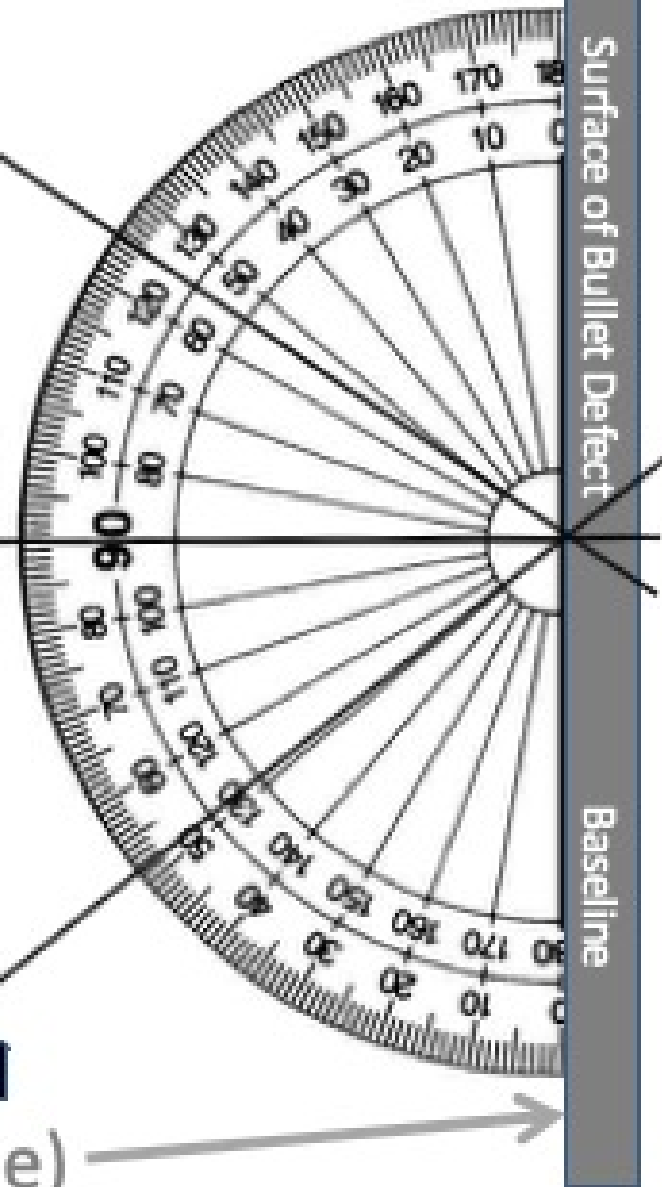
Protractor must be vertical.

Reading Elevation Angles

35° I
(125° from baseline)

90°=0°

39° I
(51 degrees from baseline)



90° (perpendicular) = 0° (Both Elevation and Azimuth)
Try it....you might just like it!!



Practical Exercises

- **#1 Practical Road AD**
 - Trajectory analysis thru trigonometry
- **#2 Mystery Shooting**
 - Trajectory and ricochet analysis thru trigonometry
- **#3 Death by Gravity**
 - Trajectories outside the box